Chapter 1

Introduction

One dimensional linear spectral properties in single fluid magnetohydrodynamics (MHD) were explored in this thesis by considering plasmas with and without mass flows. Electromagnetic field equations in vacuum are linear systems. However, by introducing magnetofluids in the system, MHD equations become highly nonlinear due to coupling between the electromagnetic and fluid properties of plasmas. In such nonlinear systems, a lot of fascinating collective phenomena will appear; chaotic motions, self-organizations, nonlinear waves, and so on. It should be stressed that even the linear problems, which describe the dynamics of small amplitude motion around an equilibrium state, also contain much more interesting behavior than a simple system of point mass, or that governed by the linear Schrödinger equation. The infinite dimensionality of the generator induces continuous spectra, and moreover, as well as neutral fluids, the generator in case of finite shear flow equilibrium becomes non-Hermitian (non-selfadjoint), which is still an unresolved problem in modern mathematics.

In addition to the above mentioned significance from mathematical physics, spectral analysis of MHD is also related to an energy research for future human lives; it is magnetic fusion. A fundamental requirement for realizing a fusion reactor is to confine a high density and high temperature[1] plasma for sufficiently long time. In order to obtain such a plasma, magnetic confinement systems have been studied, which basically utilize cyclotron motion of charged particle in a toroidal magnetic field. For the sustainment of plasma as a whole, at least the linear stability of the confined plasma is indispensable in the time scale of the plasma dynamics. Studies of such stability properties are accomplished by the spectral analysis.

[1] Actually, the highest temperature was achieved at JT-60U in 1998 at more than five hundred million degrees of centigrade, which is considered to be a few tens times higher than the core temperature of the Sun [13].

The practical magnetic confinement configurations for aiming at fusion reactors are mainly toroidal. Thus, multi-dimensional analyses of MHD stability are required. Multi-dimensional spectral properties for static plasmas are recently discussed in many literatures, e.g. for 2D [56, 150] and 3D [59]. However, the multi-dimensional stability analysis is much more difficult for the magnetized plasmas with shear flows.

In this thesis, one dimensional spectral properties are discussed rather deeply including discrete and continuous spectra in non-Hermitian operators. First, we will treat point spectra in Chap. 3 and continuous spectra in Chap. 4 for Hermitian operators (static plasmas). Although there are still many unresolved problems of MHD stability in static equilibria, recently MHD stability of equilibria with plasma flows are investigated intensively, because of the spontaneous formation of zonal flows from electrostatic turbulence [82]. Another reason is the rotational stabilization of MHD instabilities, e.g. resistive wall modes [122]. Recently, a confinement system which actively uses the flow shear for the improved confinement is even proposed [101]. Thus, the latter half of the thesis is devoted to non-Hermitian problems of sheared plasma flow. Point spectra and continuous spectra are discussed in Chap. 5 and in Chap. 6, respectively. Finally, the resonance between point spectra and continuous spectra is mathematically investigated in Chap. 7.

Let us describe the main results in each chapter with an emphasis on physics. We will firstly introduce the MHD system of equations in Chap. 2. Usually, pre-Maxwell equations for electromagnetic field, which comes from the neglect of the displacement current, are used in ideal MHD. We will discuss in detail why and how the displacement current will be omitted. We will also introduce the spectral method for the analysis of linearized MHD equations. The boundary condition is well known to affect the spectra of differential equations. Here, it is explicitly shown that the 'norm' (or scalar product) also plays an essential role for especially Hermiticity of the operator. The construction of general solution is guaranteed if the generating operator is shown to be Hermitian under any norm. Mathematically, boundary condition and norm are considered in the definition of the Hilbert space in which the formal differential operators are embedded. After all description of the mathematical background, we will summarize the spectra of incompressible MHD equations, on which the rest part of the thesis is based.

In Chap. 3, some unusual properties of the unstable point spectra are investigated [133, 45, 88, 47]. The Mercier (Suydam) criterion for localized interchange modes in stellarators does not predict instability in cases where global modes are unstable. One case is non-resonant pressure driven instabilities with low mode numbers, which become unstable even if the mode resonant surface does not exist inside the plasma column. The other case is interchange instabilities when the pressure gradient vanishes at the mode resonant surface. If the pressure becomes flat in a narrow region around the mode resonant surface, high mode number instabilities may be eliminated and the beta (ratio of plasma pressure to magnetic pressure) limit at the particular resonant surface increases. Also radial mode structure at nearly marginal beta changes significantly. Resonant unstable modes have a sequence of eigenvalues which converges to the edge of the Alfvén continuum, and the corresponding eigenfunctions tend to localize around the resonant surface. However, non-resonant unstable modes show the global structure and have a step like structure around the resonant surface even in the limit of marginal beta value. Property of the non-resonant mode and transition from the resonant to the non-resonant one are clarified with a cylindrical plasma model for a low shear stellarator with a magnetic hill.

In Chap. 4, Alfvén continuous spectrum is picked up [132]. The MHD wave is studied when two steep density gradient regions exist at surfaces of slab plasma. In such a case, it is shown that the surface Alfvén wave has two branches with nearly the same damping rates, since the steep density gradients are located closely each other. However, for the sharp boundary plasma, the surface Alfvén wave does not damp. As the density profile is relaxed, the damping rates become larger, pass via extremum, and again they become small when the scale length of the density gradients becomes extremely large. These damping rates seem consistent with behavior of magnetic fluctuations observed in the Heliotron-E pellet injection experiment.

As an example of non-Hermitian operators, we will first analyze the effect of the outer resistive conducting wall and the rigid flow of the plasma on external kink instabilities in Chap. 5 [153]. Rigid plasma flow itself will not introduce non-Hermiticity, however, the finite difference of the velocity between the resistive wall and the plasma will be regarded as a kind of discontinuous shear of the flow. We will firstly show the physical mechanism of Kelvin-Helmholtz instability in the neutral fluids, then show the correspondence of external kink mode in the cylindrical plasma with a surrounding resistive wall and a rigid plasma flow to the Kelvin-Helmholtz instability. It is shown that the resistive wall does not only affect to push back the plasma surface wave, but also pull to destabilize the external kink modes.

In Chap. 6, transient and secular behavior of interchange fluctuations is analyzed in an ambient shear flow by invoking Kelvin's method of shearing modes [143, 131]. Because of its non-Hermiticity, complex transient phenomena can occur in a shear flow system. It is shown that, for each mode, the combined effect of shear flow stretching and Alfvén wave propagation overcomes the instability driving force at sufficiently long time, and damps fluctuations of the magnetic flux for all wave numbers. On the other hand, electrostatic perturbations can be destabilized for sufficiently strong interchange drive. The time asymptotic behavior in both cases is algebraic (non-exponential).

In Chap. 7, we will construct an exact mathematical formulation of neutral fluids which contains the resonance of point spectra (diocotron modes) and continuous ones (entropy waves) [152]. It is easily understood by the finite dimensional spectral theory that the simple resonance among point spectra will cause secular growth of the modes. However, by introducing a new definition of the Hilbert space, it is shown that the resonance, which contains the energy flow from continuous spectra to point ones, may not cause secular behavior of the perturbed quantities. By considering another model, e.g. parallel dynamics, it is also shown in Ref. [84] that such secular behavior will be realized due to the inclusion of the resonance between two different continuous spectra. It means that, even if the system has no unstable eigenvalue [91], growth of the algebraic type is possible, which may cause linear instability of the system. It is pointed out in Appendix C that such kind of resonance (between continuum and point spectra) is also found in the kinetic treatment of the electrostatic oscillations (plasma oscillations).