Chapter 8

Concluding remarks

In this thesis, one dimensional linear spectral properties in incompressible single fluid magnetohydrodynamic (MHD) plasma were explored for several cases related to stability problem in magnetically confined plasmas for fusion research.

In Chap. 2, we have discussed the Hermiticity in connection with the norm of the operator in detail. Boundary conditions are well known to affect the spectra of the operators, however, in this chapter, it is explicitly described that the Hermiticity strongly depends on how we take the norm for the operator. If we may prove the Hermiticity of the operator with a certain norm, we can construct a spectral resolution under this norm. We have reviewed the basic spectra of shear Alfvén waves for static plasma (Hermitian generator) in both the slab and cylindrical geometry. Moreover, it is shown that the linearized equation of two dimensional incompressible Euler fluid can be also shown Hermitian if the ambient shear flow does not include any inflection point (Rayleigh's criterion). However, the system which contains any complex eigenvalue is not such a case.

Chapters 3 and 4 are devoted to the analysis of Hermitian operators. In Chap. 3, we have shown the analysis for non-resonant interchange instabilities (point spectra). It has been discussed why the prediction of the Mercier (Suydam) criterion for localized interchange modes does not explain some stable discharges achieved in stellarators. Usually, the destabilizing pressure gradient at the mode resonant surface makes the unstable mode to be localized. However, the local flattening of the pressure profile prohibits the localization of the unstable eigenmode even in the limit of marginal stability. With the numerical computation of the spectral ordinary differential equation, it is shown that the unstable eigenmode tends to have a step like structure at the resonant surface in this case. It is also found that the growth rate of the non-resonant mode decreases to zero without the tail formation near the beta (ratio of plasma pressure to magnetic pressure) limit like the Suydam modes.

The obtained beta limit of the non-resonant mode becomes mostly an order of magnitude higher than that obtained from Suydam criterion for the smooth pressure profile. This may explain the discrepancy of the experimental result that the beta exceeded the Mercier limit. Furthermore, we have shown the situation where the unstable non-resonant mode sets the beta limit which is lower than calculated from Mercier (Suydam) criterion. These perspectives obtained from cylindrical plasma model are also confirmed with a toroidal model for interchange modes.

In Chap. 4, we have calculated the damping of the surface Alfvén wave due to continuous spectra induced by the rapid change of equilibrium density profile in a slab geometry by means of the Laplace transformation method. As is shown in Sec. 2.6, every singular eigenfunction corresponding to the Alfvén continuous spectra has logarithmic singularity originated from the regularity of the singular point in the spectral ordinary differential equation. When we solve the problem by means of the Laplace transformation, it brings about the singularity of the Green's function in the analytically continuated Riemannian surface, which yields the damping rate (exponential damping) of the magnetic fluctuation. The results were compared with experimental observation of the pellet injection in Heliotron-E. The damping rate has a maximum in relation to the sharpness of density gradient, and numerical results show that the damping rate is consistent with experiment for an adequate physical parameters. In the sharp density limit (step function), this damped solution connects to the surface eigenmode.

Chapters 5, 6, and 7 are devoted for the spectral analyses of non-Hermitian (non-selfadjoint) operators. In Chap. 5, the effect of surrounding resistive wall on rigidly flowing plasma has been investigated. Since the whole system is not represented in the form of single evolution equation, it is difficult to construct a spectral representation. However, the detailed property and the behavior of an eigenmode (a point spectrum) are explored. By approximating the system in a slab geometry, we have estimated the effect of the resistive wall with replacing the wall current by a mirror image current. In static plasmas, the external kink mode (surface current driven instability) with surrounding resistive wall carries a smaller image current, which causes the instability in the resistive wall time scale. However, when plasma is rigidly flowing with respect to the wall, the image current is not only weak compared to the plasma surface current, but also becomes phase shifted, which yields another destabilizing mechanism similar to Kelvin-Helmholtz instability in neutral fluids. The combination of the magnitude and relative phase of the image current produces a small hump of the growth rate with respect to the wall position, which is related to the closest limit of wall position for the stabilization of external kink instability due to continuous spectrum. The dispersion relation is analytically solved by means of the perturbation method, and the wall position of the maximum growth rate is evaluated from the breakdown point of the perturbation expansion. It is predicted that, if the flow velocity is increased, the extremum of the growth rate will become closer to the plasma edge, however, the dependence of growth rate on the flow velocity is very weak (logarithmic).

In Chap. 6, transient and secular behaviors of interchange fluctuations are analyzed in an ambient shear flow by invoking Kelvin's method of shearing modes. The expansion with Kelvin's modes is shown to give general solutions in the case of linear shear flow profile. However, since it is only applicable to the system with infinite domain unfortunately, we cannot study the effect of shear flow on the point spectra. By means of this method, we have first analyzed the incompressible electromagnetic perturbations in the presence of an interchange drive and obtained the ordinary differential equation for the amplitude of the modes. All modes show asymptotic decay proportional to the inverse power of time (non-exponential) without any threshold value. This means that the interchange instabilities are always damped away at sufficiently long time due to the combined effect of the Alfvén wave propagation and the distortion of mode structure by means of the background shear flow; i.e. phase mixing effect. However, the transient behavior is not common for all modes. Fluctuations with particular wave numbers show transient amplifications which are even faster than the growth rate of interchange modes in static plasmas. These amplifications are so prominent that they may lead to the break down of the linearity of the perturbation. On the time evolution for the perturbations with purely perpendicular wave vectors to the ambient magnetic field, which do not excite the Alfvén wave, the flow shear has been also shown to have a stabilizing effect. However, the phase mixing effect alone cannot completely stabilize the interchange instabilities and the algebraic growth of the perturbation still remains. The condition for the boundedness of mode amplitude of the stream function is obtained; however, the conditions of different physical quantities do not coincide. The discrepancies originate from the fact that, in shear flow systems, different perturbations experience algebraic evolutions characterized by different powers of time, while the time evolutions for any fields are expressed in a common exponential form for static systems. This may represent a pathological aspect of the stability problem in shear flow plasmas.

In Chap. 7, the spectral theory of Rayleigh equation is demonstrated with surface wave model by invoking a proper definition of the Hilbert space. When the system is Kelvin-Helmholtz unstable with complex eigenvalues, the generator can be properly diagonalized. The generator turns out to be an infinite dimensional one correspondent to the semi-simple matrix in the finite dimensional linear space. It is shown, however, that the system contains the resonance of point spectra (corresponding to diocotron modes) and continuous spectra (corresponding to entropy waves) in the case where the equilibrium is stable to the Kelvin-Helmholtz mode (no complex eigenvalue). Simple resonance among point spectra will cause a secular growth of the mode; however, it is shown that the resonance, which contains the energy flow from continuous spectra to point ones, may not cause secularity of the perturbed quantities. It is noted that, by considering the another model, e.g. parallel dynamics of electrons, such secular behavior will be realized due to the inclusion of the resonance between two continuous spectra. It means that, even if we do not have any unstable eigenvalue, we may have growth of the algebraic type, which may cause an instability of the system.