Vector Relations

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = (\mathbf{ABC}) \tag{1}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B}) \tag{2}$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C}) \tag{3}$$

$$(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = (\mathbf{ABD})\mathbf{C} - (\mathbf{ABC})\mathbf{D} = (\mathbf{ACD})\mathbf{B} - (\mathbf{BCD})\mathbf{A} \tag{4}$$

$$\nabla\cdot(f\mathbf{A}) = \mathbf{A}\cdot\nabla f + f\nabla\cdot\mathbf{A} \tag{5}$$

$$\nabla\times(f\mathbf{A}) = \nabla f\times\mathbf{A} + f\nabla\times\mathbf{A} \tag{6}$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + \mathbf{A}\cdot\nabla\mathbf{B} + \mathbf{B}\cdot\nabla\mathbf{A} \tag{7}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \tag{8}$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + \mathbf{B}\cdot\nabla\mathbf{A} - \mathbf{A}\cdot\nabla\mathbf{B} \tag{9}$$

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \tag{10}$$

$$\nabla\times\nabla f = 0 \tag{11}$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0 \tag{12}$$

$$\mathbf{A}\cdot\nabla\mathbf{A} = \nabla\left(\frac{1}{2}A^2\right) - \mathbf{A}\times(\nabla\times\mathbf{A}) \tag{13}$$

$$\nabla\times(\mathbf{A}\cdot\nabla\mathbf{A}) = \mathbf{A}\cdot\nabla(\nabla\times\mathbf{A}) + \nabla\cdot\mathbf{A}(\nabla\times\mathbf{A}) - \{(\nabla\times\mathbf{A})\cdot\nabla\}\mathbf{A} \tag{14}$$

$$\mathbf{A}\cdot\{\mathbf{B}\cdot\nabla(\nabla f)\} = \mathbf{B}\cdot\{\mathbf{A}\cdot\nabla(\nabla f)\} \tag{15}$$

$$\mathbf{A}\cdot(\mathbf{B}\cdot\nabla\mathbf{C}) = \mathbf{B}\cdot(\mathbf{A}\cdot\nabla\mathbf{C}) - (\mathbf{A}\times\mathbf{B})\cdot(\nabla\times\mathbf{C}) \tag{16}$$

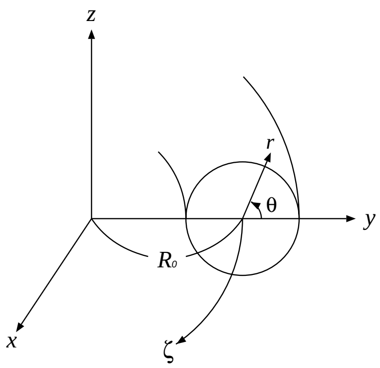
$$\mathbf{A}\cdot\nabla(\mathbf{B}\cdot\nabla f) = (\mathbf{A}\cdot\nabla\mathbf{B})\cdot\nabla f + (\mathbf{AB}:\nabla\nabla)f \tag{17}$$

$$\mathbf{A}\cdot\nabla(\mathbf{B}\cdot\nabla\mathbf{C}) = (\mathbf{A}\cdot\nabla\mathbf{B})\cdot\nabla\mathbf{C} + (\mathbf{AB}:\nabla\nabla)\mathbf{C} \tag{18}$$

$$[(\mathbf{A}\times\mathbf{B})\times\nabla]\times\mathbf{C} = \mathbf{B}\times(\mathbf{A}\cdot\nabla\mathbf{C}) - \mathbf{A}\times(\mathbf{B}\cdot\nabla\mathbf{C}) \tag{19}$$

$$(\mathbf{A}\times\nabla)\times\mathbf{B} = (\nabla\mathbf{B})\cdot\mathbf{A} - \mathbf{A}(\nabla\cdot\mathbf{B}) \tag{20}$$

Quasi-toroidal Coordinates



Definition

$$\left\{\begin{array}{l} x = (R_0 + r\cos\theta)\sin\phi \\ y = (R_0 + r\cos\theta)\cos\phi \\ z = r\sin\theta \end{array}\right. \tag{1}$$

Unit vectors

$$\left\{\begin{array}{l} \boldsymbol{e}_r = \boldsymbol{e}_x\cos\theta\sin\phi + \boldsymbol{e}_y\cos\theta\cos\phi + \boldsymbol{e}_z\sin\theta \\ \boldsymbol{e}_\theta = -\boldsymbol{e}_x\sin\theta\sin\phi - \boldsymbol{e}_y\sin\theta\cos\phi + \boldsymbol{e}_z\cos\theta \\ \boldsymbol{e}_\phi = \boldsymbol{e}_x\cos\phi - \boldsymbol{e}_y\sin\phi \end{array}\right. \tag{2}$$

Metric

$$d\ell^2 = dr^2 + r^2 d\theta^2 + (R_0 + r\cos\theta)^2 d\phi^2 \tag{3}$$

Jacobian matrix

$$\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)} = \begin{pmatrix} \cos\theta\sin\phi & \cos\theta\cos\phi & \sin\theta \\ -\dfrac{1}{r}\sin\theta\sin\phi & -\dfrac{1}{r}\sin\theta\cos\phi & \dfrac{1}{r}\cos\theta \\ \dfrac{\cos\phi}{R_0 + r\cos\theta} & -\dfrac{\sin\phi}{R_0 + r\cos\theta} & 0 \end{pmatrix} \tag{4}$$

$$\left|\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}\right| = \frac{1}{r(R_0 + r\cos\theta)} \tag{5}$$

Gradient operator

$$\nabla = \boldsymbol{e}_r\frac{\partial}{\partial r} + \boldsymbol{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \boldsymbol{e}_\phi\frac{1}{R_0 + r\cos\theta}\frac{\partial}{\partial\phi} \tag{6}$$