Stabilization effect of magnetic shear on the diocotron instability

Shigeo Kondoh, Tomoya Tatsuno, and Zensho Yoshida
Graduate School of Frontier Sciences, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

(Received 13 December 2000; accepted 7 February 2001)

The diocotron instability in a magnetized non-neutral plasma is a close cousin of the Kelvin–Helmholtz instability. A sheared magnetic field brings about coupling between the diocotron modes and the Langmuir waves that propagate along the magnetic field. The motion of electrons parallel to the magnetic field cancels the electric charge produced by the diocotron modes, resulting in stabilization of the diocotron instability.
[DOI: 10.1063/1.1371521]

I. INTRODUCTION

Recently a variety of new concepts on non-neutral plasma confinement have been proposed,[1–3] which significantly differ from the conventional Penning/Malmberg trap.[4,5] The Prototype Ring Trap (Proto-RT) experiment[1,6,7] is aimed at pure magnetic confinement of a toroidal non-neutral plasma that is not in a rigid-rotating thermal equilibrium state. In such a system, the plasma flow is generally sheared, and the diocotron instability[8] can be destabilized. The application of magnetic shear is expected to be most effective to stabilize the electrostatic modes. However, an exact stability analysis has not been completed, except for the special case of an electron beam with a relativistic speed.[9]

The physical mechanism of the diocotron instability is explained as follows[10] (see Fig. 1): When a non-neutral slab plasma has a finite thickness, a perturbation on one of the two plasma surfaces produces surface charges. The resulting perturbed electric field yields an $\mathbf{E}\times\mathbf{B}$ flow in the plasma, and the opposite surface is also perturbed. The motion of the opposite surface brings about a reciprocal perturbation, and the waves on the two surfaces couple with each other. Under certain conditions, this coupling yields a positive feedback, and the diocotron instability occurs.

When a non-neutral plasma is confined in a uniform magnetic field, the diocotron modes propagating in the perpendicular direction to the magnetic field are independent of any modes that propagate in the parallel direction. However, if the magnetic field has a shear (see Fig. 2), the wave vector may have a local parallel component $k_\parallel(x)$, and the diocotron modes interact with the parallel modes, such as the Langmuir wave or the plasma oscillation. In a cold non-neutral plasma, the surface charge perturbation produced by the diocotron modes is short-circuited by the parallel motion of charged particles, if the diocotron frequency ω_D is much smaller than the plasma frequency ω_p, i.e., when a low density plasma embedded in a strong magnetic field. Therefore, we expect that the diocotron instability is stabilized in a sheared magnetic field.

In this paper, we consider a slab plasma with a flat-top density profile and show the stabilizing effect of magnetic shear analytically. The diocotron instability is formally equivalent to the Kelvin–Helmholtz instabilities in fluids and plasmas.[11] The magnetic shear stabilization of these instabilities is of the common interest and has a variety of applications (see Sec. IV).

II. EIGENEQUATION FOR DIOCOTRON MODES IN A SHEARED MAGNETIC FIELD

A. Slab plasma model in a sheared magnetic field

We consider a slab electron plasma embedded in a sheared magnetic field (see Fig. 2). The plasma has a finite thickness 2Δ in the x-direction. We assume that all equilibrium quantities are functions of only x. We consider a sheared magnetic field such as

$$\mathbf{B}=(0,B_y(x),B_z), \tag{1}$$

where B_z is a constant. Since a non-neutral plasma has a self-electric field, there is a stationary flow that is approximately equivalent to the $\mathbf{E}\times\mathbf{B}$ drift for low densities.

The governing equations are

$$\frac{\partial n}{\partial t}+\mathbf{v}\cdot\nabla n+n\nabla\cdot\mathbf{v}=0, \tag{2}$$

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{1}{s^2}(\mathbf{E}+\mathbf{v}\times\mathbf{B}), \tag{3}$$

$$\nabla^2\phi=-n, \tag{4}$$

where n is normalized by the typical electron density $n_0(0)$, t by the inverse of the diocotron frequency $\omega_D^{-1}=\varepsilon_0 B_z/en_0(0)$ (ε_0 is the vacuum dielectric constant and e is the elementary electric charge), the spatial coordinates x, y, z by the half thickness of the slab plasma Δ, $\mathbf{v}$ by the mean flow velocity at the plasma surface $|\mathbf{v}_0(1)|$, $\mathbf{B}$ by the axial magnetic field B_z, $\mathbf{E}$ by the mean electric field at the plasma surface $|\mathbf{E}(1)|$, $s\equiv\omega_p/\omega_c=\omega_D/\omega_p$ is a dimensionless parameter, and $\mathbf{E}=-\nabla\phi$. Since we consider a low-density plasma in a strong axial magnetic field, we may assume $s\ll 1$. In this limit, we can replace Eq. (3) with

$$\mathbf{v}_\perp=\frac{-\nabla\phi\times\mathbf{B}(x)}{B(x)^2}, \tag{5}$$

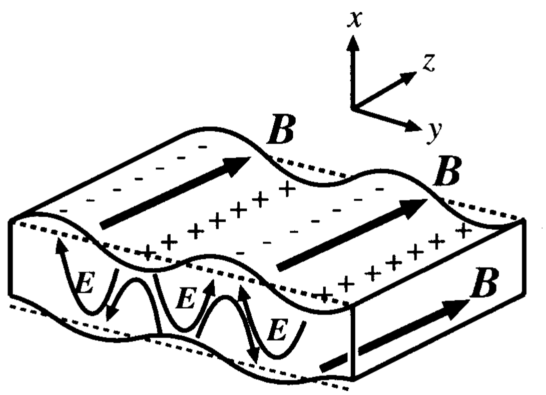

FIG. 1. A physical picture of diocotron modes in a uniform magnetic field. Perturbation on one of the two plasma surfaces produces the surface charge and causes the electrostatic field perturbation. This perturbed electric field shakes the body of the plasma itself through $E\times B$ drift, and the other surface is also perturbed. The perturbation on the latter surface in turn shakes the former one in the same way. Thus, the waves on the two surfaces couple with each other. Under certain conditions, the diocotron modes can be unstable.

$$\frac{\partial v_\parallel}{\partial t}+(\mathbf{v}\cdot\nabla)v_\parallel=\frac{1}{s^2}\nabla_\parallel\phi, \tag{6}$$

where $B(x)=\sqrt{B_y(x)^2+B_z^2}$. We are neglecting the polarization drift velocity $\mathbf{v}_p$, because $\mathbf{v}_p=O(s^2)\ll 1$. Substituting Eq. (5) into Eq. (2) and linearizing the resulting equation, we obtain

$$(\omega-k_y v_{0y}-k_z v_{0z})n_1-\frac{d}{dx}\left[\frac{B_y k_z-B_z k_y}{B^2}n_0\right]\phi_1-in_0\nabla\cdot\mathbf{v}_{\parallel 1}=0, \tag{7}$$

where we have Fourier-transformed all perturbed variables $\Psi(x,y,z,t)$ as

$$\Psi(x,y,z,t)=\Psi_0(x)+\Psi_1(x)\exp[i(\omega t-k_y y-k_z z)]. \tag{8}$$

Here we perform a coordinate transform,

$$x=x',\quad y=\frac{B_z}{B}y'+\frac{B_y}{B}z',\quad z=-\frac{B_y}{B}y'+\frac{B_z}{B}z', \tag{9}$$

on Eqs. (7). The new z'-axis is set to parallel the direction of the magnetic field, and the new y'-axis is perpendicular to both the x- and z'-directions. Equation (7) now reads as

$$(\omega-k_\perp(x)v_{\perp 0})n_1+\frac{d}{dx}\left(\frac{k_\perp(x)n_0}{B}\right)\phi_1-n_0k_\parallel(x)v_{\parallel 1}=0, \tag{10}$$

where $k_\perp(x)$ and $k_\parallel(x)$ are the wave vector, in the perpendicular and parallel directions, with respect to the magnetic field, respectively;

$$k_\perp(x)=\frac{k_y-k_zB_y}{\sqrt{1+B_y^2}}, \tag{11}$$

$$k_\parallel(x)=\frac{k_yB_y+k_z}{\sqrt{1+B_y^2}}. \tag{12}$$

Similarly, from Eqs. (4) and (6), we obtain

$$\frac{d^2\phi_1}{dx^2}-k^2\phi_1=n_1, \tag{13}$$

$$(\omega-k_\perp v_{\perp 0})v_{\parallel 1}=-\frac{k_\parallel}{s^2}\phi_1, \tag{14}$$

where $k^2\equiv k_\perp(x)^2+k_\parallel(x)^2=k_y^2+k_z^2$, which is independent of x. Substituting Eqs. (13) and (14) into Eq. (10), we obtain the eigenequation for the perturbed electrostatic potential as

$$\left(\frac{d^2\phi_1}{dx^2}-k^2\phi_1\right)+\frac{1}{\omega-k_\perp v_{\perp 0}}\frac{d}{dx}\left(\frac{k_\perp n_0}{B}\right)\phi_1+\frac{n_0k_\parallel^2}{s^2(\omega-k_\perp v_{\perp 0})^2}\phi_1=0. \tag{15}$$

B. Coupling between diocotron wave and plasma oscillation

The last term of Eq. (15) represents the coupling of the parallel dynamics and the diocotron modes. In the limit of a shearless magnetic field ($B_y=0$), the diocotron modes and plasma oscillation are decoupled, and, hence, the last term of the eigenequation (15) vanishes ($k_\parallel=0$), and (15) reduces to

$$\left(\frac{d^2\phi_1}{dx^2}-k_y^2\phi_1\right)+\frac{k_y}{\omega-k_yv_{y0}}\frac{dn_0}{dx}\phi_1=0, \tag{16}$$

which is equivalent to Rayleigh's equation for the Kelvin–Helmholtz instability.[12,13] The diocotron instability, which is thus a cousin of the Kelvin–Helmholtz instability, is caused by the non-self-adjointness brought about by the second term in Eq. (15).

In the sheared magnetic field, the diocotron modes always couple with the plasma oscillation in the direction parallel to the magnetic field. This interaction is represented by

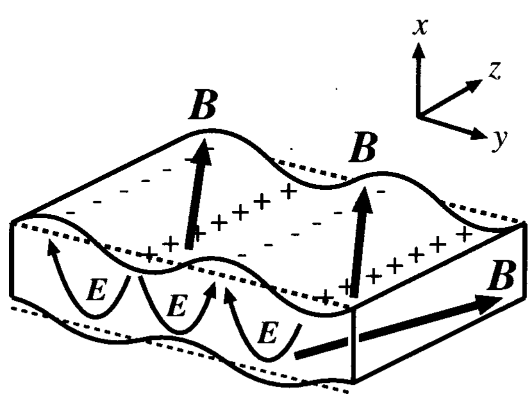

FIG. 2. A physical picture of stabilizing effect of a sheared magnetic field on diocotron modes. The wave vector almost always has a local parallel component, and the diocotron modes cannot be independent of the parallel modes, such as the Langmuir wave or the plasma oscillation. Therefore, a coupling between them is caused. In a cold non-neutral plasma, the surface charge perturbation produced by the diocotron modes is canceled by the parallel motion of charged particles. Thus, the diocotron instability is stabilized by the sheared magnetic field.

the last term in Eq. (15). The last term includes a small parameter s^2 in its denominator, so a significant change is brought about in the characteristics of the equation. Physically this means that the parallel motion of the plasma easily cancels the charge perturbation and has a strong short-circuiting effect under the condition that the time scale of the plasma oscillation is much smaller than the diocotron modes $(s=\omega_D/\omega_p \ll 1)$.

In the next section, we will show analytically that the last term has a stabilizing effect.

III. STABILIZING EFFECT DUE TO PARALLEL MOTION

A. Nonresonant frequency regime

First, we show that the diocotron modes are stabilized $(\omega_i=0)$ for wave numbers without resonance between the phase velocity and the plasma flow, that is, $\omega_r - k_\perp v_{\perp 0} \neq 0$ for all x. Multiplying Eq. (15) by ϕ^* and integrating it over $(-\infty,\infty)$, we obtain from the imaginary part,

$$\omega_i \int_{-\infty}^{\infty} \left[\frac{1}{|\omega - k_\perp v_{\perp 0}|} \frac{d}{dx}\left(\frac{k_\perp n_0}{B}\right) + \frac{2 n_0 k_\parallel^2 (\omega_r - k_\perp v_{\perp 0})}{s^2 |\omega - k_\perp v_{\perp 0}|^4} \right] |\phi_1|^2 dx = 0. \tag{17}$$

Here we used the boundary condition

$$\phi_1(\pm\infty)=0. \tag{18}$$

Since $s^2 \ll 1$ and $\omega_r - k_\perp v_{\perp 0} \neq 0$ at any point in the plasma region, we obtain $\omega_i=0$, which means stability. This mathematical treatment is the same as the standard Rayleigh's analysis.[12]

B. Dispersion relation with resonances

If the plasma has a resonant point, the analysis in Sec. III A does not apply to check whether the eigenvalues ω for Eq. (15) are real or not. In this case, we need to solve Eq. (15) directly. The eigenfunction determined by Eq. (15) is oscillatory, because the sign of the last term, which we assumed to be very large $(s \ll 1)$, is positive. If ω is not real, the real and imaginary parts of the eigenfunction have a relative phase angle of about $\pi/2$. When we consider a density profile with a sharp boundary, we have to connect both the real and imaginary parts of the eigenfunction at the plasma surfaces using the same boundary condition. If both of them have a different phase angle, this process fails, which implies that ω must be real.

The essential characteristic of this eigenvalue problem is well understood by the following simplified model. First, we neglect the second term $k^2\phi_1$ in the bracket of the first term of Eq. (15) in the plasma region $(-1,1)$, since it is much smaller than the last term when $n_0 \simeq 1$. We also assume that $k_\perp n_0/B$ jumps at $x=\pm 1$ and its variation is negligible anywhere else, i.e.,

$$\frac{d}{dx}\left(\frac{k_\perp n_0}{B}\right) = f(x)[\delta(x+1)-\delta(x-1)], \tag{19}$$

where $f(x)$ is a given finite function. Furthermore, we assume

$$\frac{n_0(x) k_\parallel(x)^2}{s^2} \equiv a^2 = \text{const} \gg 1, \tag{20}$$

$$k_\perp(x) v_{\perp 0}(x) = x. \tag{21}$$

Under these assumptions, Eq. (15) reduces to

$$\frac{d^2\phi_1}{dx^2} + \frac{a^2}{(\omega - x)^2}\phi_1 = 0 \quad (|x|<1), \tag{22}$$

$$\frac{d^2\phi_1}{dx^2} - k^2\phi_1 = 0 \quad (|x|>1), \tag{23}$$

with the boundary condition, Eq. (18), and the jump conditions

$$\frac{d\phi_1}{dx}(-1+\epsilon) - \frac{d\phi_1}{dx}(-1-\epsilon) = -\frac{f(-1)}{\omega+1}\phi_1(-1), \tag{24}$$

$$\frac{d\phi_1}{dx}(1+\epsilon) - \frac{d\phi_1}{dx}(1-\epsilon) = \frac{f(1)}{\omega-1}\phi_1(1). \tag{25}$$

The general solution to Eqs. (22) and (23) is given by $(k>0)$

$$\phi_1(x) = \begin{cases} \phi_{\mathrm{I}} = C_1 e^{kx} \quad (x<-1), \\ \phi_{\mathrm{II}} = C_2(\omega - x)^{(1-\sqrt{1-4a^2})/2} \\ \qquad + C_3(\omega - x)^{(1+\sqrt{1-4a^2})/2} \\ \quad \simeq C_2(\omega - x)^{(1-2ai)/2} \\ \qquad + C_3(\omega - x)^{(1+2ai)/2} \quad (|x|<1), \\ \phi_{\mathrm{III}} = C_4 e^{-kx} \quad (x>1), \end{cases} \tag{26}$$

under the boundary condition (18). Substituting Eq. (26) into the jump conditions (24) and (25) gives

$$C_2\left[-\left(\frac{1}{2}-ai\right)(\omega+1)^{-(1+2ai)/2} + \left(\frac{f(-1)}{\omega+1} - k\right) \times (\omega+1)^{(1-2ai)/2}\right] + C_3\left[-\left(\frac{1}{2}+ai\right) \times (\omega+1)^{(-1+2ai)/2} + \left(\frac{f(-1)}{\omega+1} - k\right)(\omega+1)^{(1+2ai)/2}\right] = 0, \tag{27}$$

$$C_2\left[-\left(\frac{1}{2}-ai\right)(\omega-1)^{-(1+2ai)/2} + \left(\frac{f(1)}{\omega-1} + k\right) \times (\omega-1)^{(1-2ai)/2}\right] + C_3\left[-\left(\frac{1}{2}+ai\right) \times (\omega-1)^{(-1+2ai)/2} + \left(\frac{f(1)}{\omega-1} + k\right)(\omega-1)^{(1+2ai)/2}\right] = 0. \tag{28}$$

If Eqs. (27) and (28) have nontrivial solutions for C_2 and C_3, the following relation must be satisfied:

$$\left[-\left(\frac{1}{2}-ai\right)(\omega+1)^{-(1+2ai)/2}+\left(\frac{f(-1)}{\omega+1}-k\right)(\omega+1)^{(1-2ai)/2}\right]$$
$$\times\left[-\left(\frac{1}{2}+ai\right)(\omega-1)^{(-1+2ai)/2}+\left(\frac{f(1)}{\omega-1}+k\right)(\omega-1)^{(1+2ai)/2}\right]-\left[-\left(\frac{1}{2}+ai\right)(\omega+1)^{(-1+2ai)/2}\right.$$
$$\left.+\left(\frac{f(-1)}{\omega+1}-k\right)(\omega+1)^{(1+2ai)/2}\right]\times\left[-\left(\frac{1}{2}-ai\right)(\omega-1)^{-(1+2ai)/2}+\left(\frac{f(1)}{\omega-1}+k\right)(\omega-1)^{(1-2ai)/2}\right]=0. \tag{29}$$

Equation (29) is the dispersion relation. We can show that ω is real for Eq. (29). Substituting $\omega=\omega_r+i\omega_i$ into Eq. (29), we obtain

$$\left[(a+k\omega_i)^2-k^2\omega_r^2+\left(k+\frac{1}{2}\right)^2+f(-1)f(1)-\left(k+\frac{1}{2}\right)(f(-1)+f(1))\right.$$
$$\left.+k\omega_r(f(-1)-f(1))+i(a+k\omega_i)(f(-1)-f(1)-2k\omega_r)\right](\omega+1)^{2ai}$$
$$=\left[(a-k\omega_i)^2-k^2\omega_r^2+\left(k+\frac{1}{2}\right)^2+f(-1)f(1)-\left(k+\frac{1}{2}\right)(f(-1)+f(1))\right.$$
$$\left.+k\omega_r(f(-1)-f(1))-i(a-k\omega_i)(f(-1)-f(1)-2k\omega_r)\right](\omega-1)^{2ai}, \tag{30}$$

where $\omega_r=\mathrm{Re}\,\omega$ and $\omega_i=\mathrm{Im}\,\omega$. Taking the absolute number of Eq. (30) gives

$$\frac{|A_1|}{|A_2|}=\exp\left[2a\,\arg\left(\frac{\omega+1}{\omega-1}\right)\right], \tag{31}$$

where

$$A_1=(a+k\omega_i)^2-k^2\omega_r^2+\left(k+\frac{1}{2}\right)^2+f(-1)f(1)$$
$$-\left(k+\frac{1}{2}\right)(f(-1)+f(1))+k\omega_r(f(-1)-f(1))$$
$$+i(a+k\omega_i)(f(-1)-f(1)-2k\omega_r), \tag{32}$$

$$A_2=(a-k\omega_i)^2-k^2\omega_r^2+\left(k+\frac{1}{2}\right)^2+f(-1)f(1)$$
$$-\left(k+\frac{1}{2}\right)(f(-1)+f(1))+k\omega_r(f(-1)-f(1))$$
$$-i(a-k\omega_i)(f(-1)-f(1)-2k\omega_r). \tag{33}$$

If $\omega_i>0$, the left-hand side of Eq. (31) is greater than unity, while the right-hand side is less than unity. Therefore, $\omega_i>0$ cannot be satisfied. If $\omega_i<0$, the left-hand side of Eq. (31) is less than unity, while the right-hand side is greater than unity. Therefore, $\omega_i<0$ cannot be satisfied. Thus $\omega_i=0$, which means stability. If $\omega_i\neq 0$, the eigenfunctions of the three regions ϕ_{I}, ϕ_{II}, and ϕ_{III} cannot be connected properly at $x=\pm 1$. We can also show that $|\omega|>1$ for the point spectrum.

The eigenfunctions given by Eq. (26) are shown in Fig. 3. All ω are real in this figure. The eigenfunctions are oscillatory in $(-1,1)$, because the sign of the second term of Eq. (22) is positive. If ω is not real, ϕ_1 must be also nonreal. Figure 4 shows a nonreal solution to Eqs. (22) and (23). As we can see from this figure, the relative phase angle between the real part and the imaginary part is about $\pi/2$. This prevents both of them from connecting properly at $x=\pm 1$.

IV. SUMMARY

We have shown that the magnetic shear has a strong stabilizing effect on the diocotron instability. The fluid motion parallel to the magnetic field short-circuits the charge perturbation of the diocotron modes. The scaling parameter is $s\equiv\omega_D/\omega_p$. Since the time scale of the parallel motion is $\sim\omega_p^{-1}$, the condition $s\ll 1$ enables the parallel motion of the plasma to cancel the perturbed charge sufficiently. Typical non-neutral plasmas in laboratories satisfy this condition.

Mathematically the last term of the eigenequation (15) prohibits nonreal eigenvalues, because the last term makes the eigenfunction oscillatory. If $\omega\notin\mathcal{R}$, the relative phase angle between the real and imaginary parts of the eigenfunction is about $\pi/2$. This phase angle disables both the real and imaginary parts of the eigenfunction to satisfy the jump conditions (24) and (25) simultaneously.

The diocotron instability is formally equivalent to the Kelvin–Helmholtz instability.[11] The present analysis of the effect of the parallel dynamics has, thus, a close analogy with the study of the Kelvin–Helmholtz instability in a sheared magnetic field[14] and auroras with the longitudinal fluctuating electric field.[15,16] In Ref. 14, Idomura *et al.* discussed the importance of the magnetic shear stabilization of the

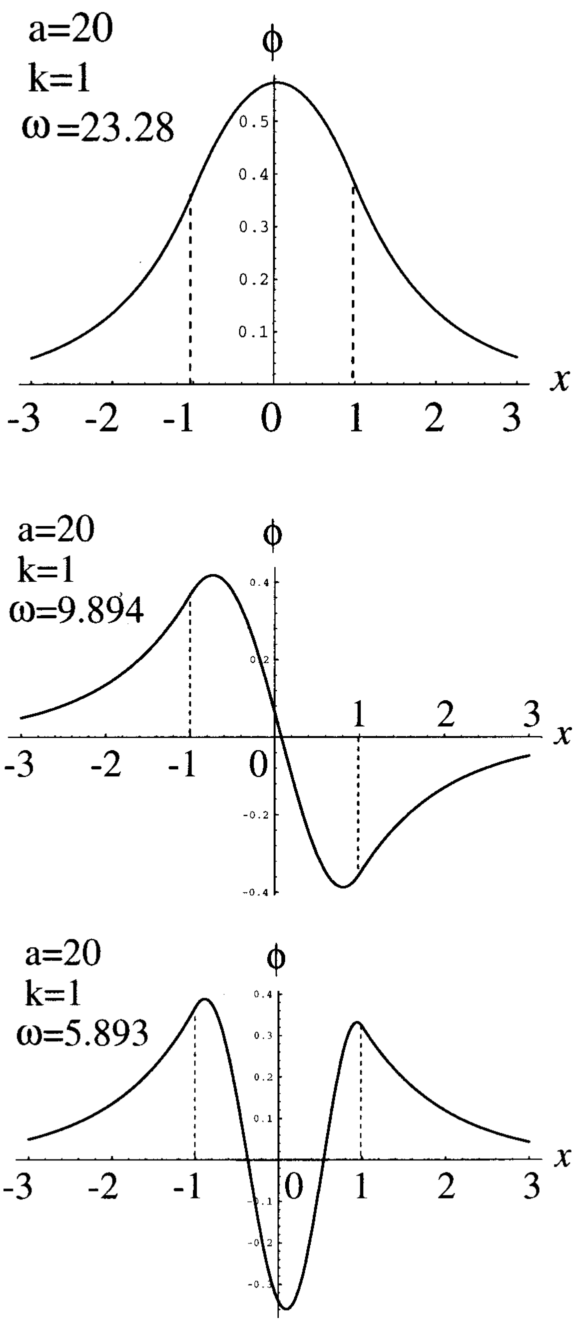

FIG. 3. Eigenfunctions given by Eq. (26). All eigenvalues are real, and no jumps at the plasma surfaces are considered in this figure. The eigenfunctions are oscillatory in the plasma region, because the sign of the second term of Eq. (22) is positive.

Kelvin–Helmholtz instability in connection with the $E\times B$ zonal flow in high-temperature tokamak plasmas.[14] Their scaling parameter is $\lambda_{\mathrm{De}}/v_{te}$ (λ_{De} is the Debye length and v_{te} is the electron thermal velocity), and they reported that the parallel Landau resonance is effective to stabilize the Kelvin–Helmholtz instability. In Refs. 15 and 16, Thompson and Satyanarayana introduced the parallel motion of electrons into their analyses to explain the field-aligned electric fields and currents in the shear-flow region of the magnetosphere. They reported that the compressional energy of electrons that comes from the parallel dynamics is the cause of stabilization of the Kelvin–Helmholtz instability.

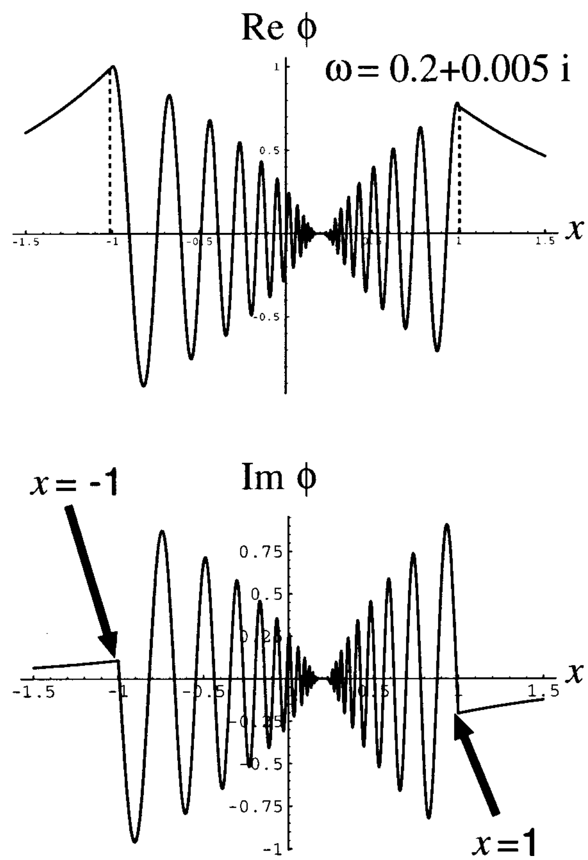

FIG. 4. A complex solution to Eqs. (22) and (23) for imaginary ω. The real part and the imaginary part of ϕ_1 have phase displacement of about $\pi/2$. This prevents both the real and imaginary parts of ϕ_1 from connecting properly at $x=\pm 1$.

Finally we note that our analysis is based on a modal approach which, however, may not be complete for non-Hermitian systems.[17–19] There remains a possibility of secular algebraic behavior, although we have shown that there are no exponentially unstable modes. This problem will be discussed elsewhere.

ACKNOWLEDGMENTS

The authors are obliged to Dr. F. Volponi for his useful discussions.

This work was supported by a Grant in-Aid for Scientific Research from the Japanese Ministry of Education, Science and Culture No. 09308011 and No. 12780353.

[1]Z. Yoshida, Y. Ogawa, J. Morikawa *et al.*, *Non-Neutral Plasma Physics III*, AIP Conf. Proc. 498 (American Institute of Physics, Melville, NY, 1999), p. 397.
[2]S. Robertson and B. Walch, Phys. Plasmas **7**, 2340 (2000).
[3]K. Avinash, Phys. Plasmas **1**, 3731 (1994).
[4]J. H. Malmberg and J. S. deGrassie, Phys. Rev. Lett. **35**, 577 (1975).
[5]J. H. Malmberg and C. F. Driscoll, Phys. Rev. Lett. **44**, 654 (1980).
[6]Z. Yoshida, Y. Ogawa, J. Morikawa, M. Hashimoto, T. Hirotsu, H. Kakuno, S. Kondo, C. Nakashima, and K. Takemura, *Extended Synopses*, 17th IAEA Fusion Energy Conference (International Atomic Energy Agency, Vienna, 1998), IAEA-CN-69/ICP/10(R), p. 248.
[7]S. Kondoh and Z. Yoshida, Nucl. Instrum. Methods Phys. Res. A **382**, 561 (1996).
[8]R. C. Davidson, *Physics of Nonneutral Plasmas* (Addison-Wesley, Redwood City, CA, 1990).

[9]E. Ott and J. M. Wersinger, Phys. Fluids **23**, 324 (1980).
[10]W. Knauer, J. Appl. Phys. **37**, 602 (1966).
[11]C. F. Driscoll and K. S. Fine, Phys. Fluids B **2**, 1359 (1990).
[12]S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Clarendon, Oxford, 1961).
[13]A. Hasegawa, *Plasma Instabilities and Nonlinear Effects* (Springer-Verlag, Berlin, 1975), p. 129.
[14]Y. Idomura, M. Wakatani, and S. Tokuda, Phys. Plasmas **7**, 3551 (2000).
[15]W. B. Thompson, J. Geophys. Res. A **88**, 4805 (1983).
[16]P. Satyanarayana, Y. C. Lee, and J. D. Huba, J. Geophys. Res. A **92**, 8813 (1987).
[17]L. N. Trefethen, A. E. Trefethen, S. C. Reddy, and T. A. Driscoll, Science **261**, 578 (1993).
[18]G. D. Chagelishvili, A. D. Rogava, and I. N. Segal, Phys. Rev. E **50**, R4283 (1994).
[19]S. M. Mahajan and A. D. Rogava, Astrophys. J. **518**, 814 (1999).