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Vortex solitons: Mass, energy, and angular momentum bunching in relativistic electron-positron plasmas

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It is shown that the interaction of large amplitude electromagnetic waves with a hot electron-positron (e-p) plasma (a principal constituent of the universe in the MeV epoch) leads to a bunching of mass, energy, and angular momentum in stable, long-lived structures. Electromagnetism in the MeV epoch, then, could provide a possible route for seeding the observed large-scale structure of the universe.

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I. INTRODUCTION

It is widely believed that the currently observed large-scale structure of the universe (the clusters and superclusters of galaxies) grew gravitationally out of small density fluctuations [1]. The imprint of this density variation in the early universe was left on the cosmic microwave background radiation in the form of spatial temperature fluctuations. The gravitational origin, however, can be only a part of the story; the gravity can enhance, but it cannot produce these fluctuations. The quest for the physical process(es) which produced the initial matter-density fluctuations has led to the emergence of the following two leading mechanisms: inflation and topological defects [2,3]. Of these the former is, perhaps, the most thoroughly investigated. According to this mechanism there exists, in the evolution of the universe, an early inflationary period in which the universe expands so rapidly (exponentially) that quantum fluctuations become trapped in the expansion. By the end of the inflation, therefore, small irregularities covering a wide range of length scales permeate the entire universe. Gravitational instability then acts on these small initial irregularities, and enhances the concentration of matter from which galaxies and clusters of galaxies eventually emerge [4]. The theory of such processes is, by no means, complete, and much needs to be done to determine whether tiny quantum fluctuations can provide a strong enough template for gravitational condensation to finally create the structures that we observe today. Cosmologists, usually, rely on the speculated existence of nonbaryonic dark matter to augment the gravitational force to aid and accelerate the structure formation. The question is far from settled.

It is natural, then, to look elsewhere for the source of the seed density ''fluctuations.'' An obvious possibility is to explore if electromagnetic interactions taking place in a plasma (known to be the source of a whole variety of linear as well as nonlinear waves) can cause the required density perturbations. In the standard cosmological model of the hot Universe (the Big Bang model), it is estimated that temperatures as high as $T\sim 10^{10}$ K~ 1 MeV prevail up to times of ~ 1 s ($t\approx 1$ sec) after the Big Bang. In this epoch, the main constituents of the Universe are photons, neutrinos, and antineutrinos, and e-p pairs [1,4]. As the plasma cools down the annihilation process $e^{+}+e^{-}\rightarrow\gamma+\gamma$ dominates, and the $e^{+}e^{-}$ pair concentration goes down. Since the equilibration rates are fast in comparison with the changes in plasma parameters, an equilibrium e-p plasma should be present in the MeV epoch of the early Universe. It is this plasma-dominated era in which we will seek the seeds for future structure formation.

Relativistic e-p plasmas were investigated quite extensively [5]. Tajima and Taniuti [6] suggested that collective processes in these plasmas could lead to interesting consequences for structure formation. In the e-p plasma of the early Universe, localized low frequency electromagnetic (EM) waves ($\hbar\omega\ll T$) could propagate as an envelope soliton due to the interaction with sound waves. Plasma density variations related to these solitons could potentially be useful toward structure formation in the Universe. However, the analysis [6] was based on a one-dimensional formulation, and the corresponding soliton solutions are likely to be unstable in higher dimensions. Berezhiani and Mahajan [7] argued that in the MeV epoch of the Universe, although e-p pairs form the dominant constituent of the plasma, a minority population of heavy ions is also present due to the baryon asymmetry. They were able to show that, under appropriate conditions [when the plasma is transparent, i.e., $\omega\gg\omega_e$, where ω (ω_e) is the pulse (plasma) frequency], the resulting e-p–ion plasma supports the propagation of stable, nondiffracting, and nondispersing EM pulses (light bullets) with a large density bunching. It was further shown in Ref. [8] that these bullets are exceptionally robust: they can emerge from a large variety of initial field distributions, and are remarkably stable. Note that the characteristic dimensions of such matter-filled light pulses are proportional to the electron to ion density ratio, and tend to be considerably larger than the skin depth ($\lambda=c/\omega_e$). The implication is that when one deals with EM structures whose characteristic dimensions (of the spatiotemporal inhomogeneities) are of the order of the skin depth, the baryon asymmetry affects can be safely neglected, and the dynamical system can be assumed as a pure electron-positron plasma.

In the present paper we examine the propagation of strong EM radiation in a hot pure e-p plasma, with the explicit aim of finding soliton-type solutions. The plasma is assumed to be transparent. We demonstrate that the dynamics of the EM field envelope is governed by a generalized nonlinear Schrödinger equation (NSE) with a defocusing nonlinearity. In one dimension, this equation admits dark soliton solutions, while in two dimensions, the so called vortex soliton solutions are also possible.

Dark solitons exist as dips on a continuous-wave background field. Being stable in one dimension, they appear as dark-stripe solitary waves in bulk medium. However, such stripes are unstable to transverse modulations, which results in the induced generation of vortices with alternating polarities.

Vortex solitons, which are the most fundamental two-dimensional (2D) soliton solutions of NSE's with an angular 2π phase ramp, appear as local dark minima in an otherwise bright background. Vortex solitons were recently observed in materials with a defocusing optical nonlinearity—the dynamics of laser beams in these materials is generally described by the NSE [9]. Since electromagnetic vortices carry angular momentum that is conserved during propagation, the generation of vortex solitons in an *e*-*p* plasma is a potent mechanism for creating domains with definite angular momenta, even out of an initial field distribution devoid of angular momentum. To keep the total angular momentum at zero, domains of equal and opposite angular momenta must be created in pairs.

II. FORMULATION

We use the following set of relativistic hydrodynamic equations in dimensionless form [7]:

$$\frac{\mathrm{d}_\pm}{\mathrm{d}t}(G^\pm\gamma^\pm)-\frac{1}{n^\pm}\frac{\partial}{\partial t}P^\pm=\mp\boldsymbol{v}^\pm\cdot\frac{\partial\boldsymbol{A}}{\partial t}\mp(\boldsymbol{v}^\pm\cdot\boldsymbol{\nabla}\phi), \quad (1)$$

$$\frac{\mathrm{d}_\pm}{\mathrm{d}t}(G^\pm\boldsymbol{p}^\pm)+\frac{1}{n^\pm}\boldsymbol{\nabla}P^\pm=\mp\frac{\partial\boldsymbol{A}}{\partial t}\pm[\boldsymbol{v}^\pm\times(\boldsymbol{\nabla}\times\boldsymbol{A})]\mp\boldsymbol{\nabla}\phi, \quad (2)$$

$$\frac{\partial n^\pm}{\partial t}+\boldsymbol{\nabla}\cdot(n^\pm\boldsymbol{v}^\pm)=0, \quad (3)$$

along with the field equation (in the Coulomb gauge $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$)

$$\frac{\partial^2}{\partial t^2}\boldsymbol{A}-\boldsymbol{\nabla}^2\boldsymbol{A}+\frac{\partial}{\partial t}\boldsymbol{\nabla}\phi+(n^-\boldsymbol{v}^--n^+\boldsymbol{v}^+)=\boldsymbol{0}, \quad (4)$$

where $\boldsymbol{p}^\pm=\gamma^\pm\boldsymbol{v}^\pm$ with factor $\gamma^\pm=[1+(\boldsymbol{p}^\pm)^2]^{1/2}$; $d_\pm/dt=\partial/\partial t+\boldsymbol{v}_\pm\cdot\boldsymbol{\nabla}$ is the comoving derivative; and $G^\pm=K_3(1/T^\pm)/K_2(1/T^\pm)$, with K_n nth order modified Bessel functions of the second kind. The superscript labels the particles, electrons $(-)$, and positrons $(+)$, respectively. In these equations the time and space variables are in units of the electron plasma frequency $\omega_e=(4\pi e^2 n_0/m_e)^{1/2}$, and the collisionless skin depth c/ω_e, respectively, the field potentials $(\phi, \boldsymbol{A})$ are in units of $m_e c^2/e$, and the relativistic momentum vector $\boldsymbol{p}^\pm$ is in units of $m_e c$. The particle number density $n^\pm$ is normalized by the equilibrium density $n_0=n_0^\pm$, and the plasma temperature $(T^\pm)$ is measured in units of $m_e c^2$. The pressure $P^\pm=n_\mathrm{r}^\pm T^\pm$, where $n_\mathrm{r}^\pm$ is the density in the rest frame of the fluid element $(n_\mathrm{r}^\pm=n^\pm/\gamma^\pm)$. The function $G(z)$ defines the ''effective'' temperature dependent mass of the particles, and has the following limiting expressions: $G\approx 1+5/2z$ for $z\gg 1$, and $G\approx 4/z$ if $z\ll 1$.

From Eqs. (1)–(3) it is straightforward to derive the adiabatic equation of state [10]:

$$\frac{n^\pm/T^\pm}{\gamma^\pm K_2(1/T^\pm)}\exp(-G^\pm/T^\pm)=\mathrm{const}, \quad (5)$$

which, at nonrelativistic temperature $(T^\pm\ll 1)$, reduces to the standard adiabatic relation $[n_\mathrm{r}^\pm/(T^\pm)^{3/2}=\mathrm{const}]$ for an ordinary gas. In the ultrarelativistic limit $[T^\pm\gg 1]$, as expected, Eq. (5) describes the photon gas $[n_\mathrm{r}^\pm/(T^\pm)^3=\mathrm{const}]$. In the ultrarelativistic case, one should take into account the radiative pressure $P_R=\sigma T^4$ $(\sigma=\pi/45\hbar^3c^3)$. For simplicity we neglect this less important effect for the current considerations. Note that in the MeV epoch, the plasma temperature $T^\pm\approx m_e c^2$ (i.e., $z\approx 1$) and $G\approx 4$, leading to an effective mass of *e*-*p* pairs of $m_\mathrm{eff}\sim 4m_e$. Since the particle masses are just a few times larger than their rest mass at these temperatures, the *e*-*p* plasma can be considered as a two component fluid rather than a photon gas.

We consider the propagation of circularly polarized EM wave with a mean frequency ω, and a mean wave number k along the z axis. The choice of circular polarization is not restrictive; it simplifies the analysis by preventing harmonic generation. The vector potential can be represented as

$$\boldsymbol{A}_\perp=\frac{1}{2}(\boldsymbol{x}+\mathrm{i}\boldsymbol{y})A_\perp(\boldsymbol{r}_\perp,z,t)\exp(ikz-i\omega t)+\mathrm{c.c.}, \quad (6)$$

where $A_\perp$ is a slowly varying function of $\boldsymbol{r}$ and t $(k\gg\boldsymbol{\nabla}, \omega\gg\partial_t)$. The unit vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ define two mutually perpendicular axes in the plane normal to the direction of wave propagation. The Coulomb gauge condition leads to the relation $A_z=(i/k)(\boldsymbol{\nabla}_\perp\cdot\boldsymbol{A}_\perp)\ll A_\perp$. Consequently the effects related to A_z will turn out to be negligibly small. We shall now follow standard methods to analyze the system. In the slowly varying amplitude approximation, the transverse, high-frequency component of the equation of motion yields the simple relation between the particle momentum and the vector potential [7]:

$$\boldsymbol{p}_\perp^\pm G^\pm=\mp\boldsymbol{A}_\perp. \quad (7)$$

The low frequency motion of the plasma is driven by the ponderomotive pressure $[\sim(\boldsymbol{p}^\pm)^2]$ of the high frequency EM field, and it does not depend on the sign of the particles' charge. If we assume that in equilibrium the electron and positron fluids have equal temperatures $(T_0^\pm=T_0)$, their effective masses will also be equal $(G^\pm=G)$, and the radiation pressure will impart equal low frequency momenta to both fluids, allowing the possibility of overall density changes without producing charge separation. The charge neutrality conditions $n^-=n^+=N$, $\phi=0$ will be assumed in the rest of this paper. It is also evident that the symmetry between the two fluids keeps their temperatures always equal ($T^{\pm}=T$) if they were equal initially.

A considerable simplification results when we invoke the wide beam approximation [11]. We assume that the longitudinal variation of the field envelope is much stronger than the transverse variation, i.e., L_z, the characteristic length along the propagation direction, is much shorter than $L_\perp$, the characteristic length in the transverse plane. This approximation, coupled with charge neutrality, allows us to extract, from Eqs. (1) and (2), the following, leading order description for the low frequency response: the equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}Gp+\frac{1}{N}\frac{\partial}{\partial z}\frac{NT}{\gamma}=-\frac{1}{2\gamma G}\frac{\partial|A_\perp|^2}{\partial z}, \tag{8}$$

and the ‘‘energy’’ conservation equation

$$\frac{\mathrm{d}}{\mathrm{d}t}G\gamma-\frac{1}{N}\frac{\partial}{\partial t}\frac{NT}{\gamma}=\frac{1}{2\gamma G}\frac{\partial|A_\perp|^2}{\partial t}. \tag{9}$$

Here we have used the condition that the ponderomotive pressure gives equal longitudinal momenta to both electrons and positrons ($p_z^{\pm}=p$). Note that the assumed circular polarization of the EM field insures that the relativistic factor γ does not depend on the ‘‘fast’’ time ($1/\omega$) scale; it can be written as

$$\gamma=\left[1+\frac{|A_\perp|^2}{G^2}+p^2\right]^{1/2}. \tag{10}$$

Substituting Eqs. (6) and (7) into Eq. (4), we find that the slowly varying amplitude $A_\perp$ must satisfy

$$2\mathrm{i}\omega(\partial_t+v_g\partial_z)A_\perp+\boldsymbol{\nabla}_\perp^2A_\perp+(\partial_z^2-\partial_t^2)A_\perp+(\omega^2-k^2)A_\perp-\frac{2N}{\gamma G}A_\perp=0, \tag{11}$$

where v_g denotes the group velocity of the carrier waves; $v_g=\mathrm{d}\omega/\mathrm{d}k=k/\omega$.

We are still not quite done with simplifying assumptions. We seek solutions which vary slowly with time in a frame comoving with the wave, that is, in a frame propagating with the group velocity v_g. The transformations $\xi=z-v_gt$ and $\tau=t$, with the condition $v_g\partial_\xi\gg\partial_\tau$, help implement this approximation. Equations (8) and (9) can now be combined to derive

$$\frac{\partial}{\partial\xi}[G(\gamma-v_gp)]=0; \tag{12}$$

the implied constant of motion is to be determined from the boundary conditions. We demand p and $A_\perp$ to be zero at infinite ξ, but allow them to be finite as $r_\perp\to\infty$. Integrating Eq. (12) leads to (T_0 is the particle temperature at infinity),

$$G(T)(\gamma-v_gp)=G_0(T_0), \tag{13}$$

which is readily solved for an explicit expression for the longitudinal momentum in terms of the transverse vector potential,

$$p=v_g\gamma_g^2\frac{G_0}{G}\left[1-\frac{1}{G_0v_g\gamma_g}(\gamma_g^2G_0^2-G^2-|A_\perp|^2)^{1/2}\right], \tag{14}$$

where $\gamma_g=1/(1-v_g^2)^{1/2}$ is the ‘‘effective relativistic factor’’ associated with the group velocity of the wave; it is not to be confused with the particle γ. The continuity equation can be similarly integrated to determine the particle density [after using Eq. (14) for p]:

$$\frac{N}{\gamma G}=\frac{v_g\gamma_g}{(\gamma_g^2G_0^2-G^2-|A_\perp|^2)^{1/2}}. \tag{15}$$

Substituting Eq. (15) into Eq. (11), we obtain the following nonlinear Schrödinger equation for the complex amplitude $A_\perp$:

$$2i\omega\partial_\tau A_\perp+\boldsymbol{\nabla}_\perp^2A_\perp+\frac{1}{\gamma_g^2}\partial_\xi^2A_\perp+\frac{2}{G_0}\left(1-\frac{v_g\gamma_gG_0}{(\gamma_g^2G_0^2-G^2-|A_\perp|^2)^{1/2}}\right)A_\perp=0, \tag{16}$$

where the wave frequency ω satisfies the dispersion relation $\omega^2=k^2+2/G_0$, implying that the parameter $\gamma_g=\omega\sqrt{G_0/2}$ [$\gamma_g=(\omega/\omega_e)\sqrt{G_0/2}$ in physical quantities]. A set comprising Eq. (16), and the equation of state (5) [in which relation (15) could be easily incorporated] constitutes a complete description of the dynamics of strong EM waves in relativistic e-p plasma in the wide beam approximation.

We remind the reader that Eq. (16) was derived under the assumption $\partial_\xi\gg\boldsymbol{\nabla}_\perp$ (i.e., $L_z\ll L_\perp$). In spite of this, for a highly transparent plasma ($\gamma_g\gg1$) the second, ‘‘diffractive,’’ term can be of the same order or even greater than the third, ‘‘dispersive,’’ term. For this paper, we will not attempt the general solutions of this quite complicated set of equations; we will simply deal with waves for which the plasma is so highly transparent that the diffractive term dominates. Using $\gamma_gG_0\gg G$, and neglecting the dispersive term, the NSE simplifies to

$$\mathrm{i}\partial_\tau A_\perp+\frac{1}{2}\boldsymbol{\nabla}_\perp^2A_\perp-2\left[\left(1-\frac{|A_\perp|^2}{\gamma_g^2G_0^2}\right)^{-1/2}-1\right]A_\perp=0, \tag{17}$$

where the following renormalizations are used: $\tau/2\omega G_0\to\tau$ and $\boldsymbol{r}_\perp/\sqrt{2G_0}\to\boldsymbol{r}_\perp$.

The vector potential $|A_\perp|$ is restricted from above by the condition $|A_\perp|<\gamma_gG_0$. This restriction is necessary for the validity of the hydrodynamic treatment for the particles. For larger amplitudes, the electromagnetic waves are overturned, causing a multistream motion of the plasma requiring a kinetic description. Note that despite the upper bound on the amplitude of the vector potential, the EM field can be still relativistically strong, i.e., the normalized $|A_\perp| \gg 1$, since $\gamma_g \gg 1$.

III. STATIONARY SOLUTIONS

In the NSE derived above the diffractive and nonlinear terms have opposite signs and as a consequence Eq. (17) does not admit transversely localized solutions (also called bright solitons). Any localized initial EM field, therefore, will undergo transverse spreading during propagation. The NSE with a defocusing nonlinearity can, however, support stationary structures with asymptotically (at infinity) nonvanishing fields. Dark solitons in one dimensions, and vortex solitons in two dimensions, are the fundamental representatives of such solutions. In the extreme low amplitude limit, $|A_\perp| \ll \gamma_g G_0$, Eq. (17) reduces to a NSE with a cubic nonlinearity. In one-dimensional geometry we have

$$\mathrm{i}\frac{\partial A_\perp}{\partial \tau} + \frac{1}{2}\frac{\partial^2 A_\perp}{\partial x^2} - \frac{1}{\gamma_g^2 G_0^2}|A_\perp|^2 A_\perp = 0. \tag{18}$$

This equation is exactly integrable via the inverse scattering method [12], and its one-soliton solution can be written as [9]

$$A_\perp(x,\tau) = \gamma_g G_0 A_0(\alpha \tanh\Theta + i\beta)e^{-iA_0^2\tau}, \tag{19}$$

where

$$\Theta = \alpha A_0(x - \beta A_0 \tau). \tag{20}$$

Here A_0 is a measure of the asymptotic fields at the spatial infinity, and α and β are constants with $\alpha^2+\beta^2=1$. The solution, with a nonzero value at the center of the dip, is termed the ''gray soliton'' to distinguish it from the ''black soliton'' (zero amplitude at the dip) corresponding to $\beta=0$. Dark solitons of this class of NSE's do not have any threshold values for their excitation, unlike bright solitons (of the appropriate equations), which do. In other words, dark solitons can be created by an arbitrary small initial dip on a homogeneous background.

In two transverse dimensions, a dark soliton represents a dark stripe imposed on a homogeneous bright background. It is well known that such a stripe is unstable to transverse, long wavelength modulations [13]. The instability causes the stripe to split into a sequence of vortex solitons of alternative polarities [14]. The vortices are dark holes on a bright background, with a nested phase dislocation of the order $m = \pm 1, \pm 2, \ldots$ at their core.

Vortex soliton solutions of the NSE were first suggested by Pitaevskii [15] as topological excitations in an imperfect Bose gas in the superfluids. The ability of some electromagnetic systems (like the e-p plasma) to simulate fluid dynamical phenomena (like vortex formation) can be demonstrated by applying the Madelung transformation $A_\perp = \sqrt{\rho}\exp(i\psi)$ to the defining equations. The transformation converts the original set to one that is similar to the fluid hydrodynamic equations with a fluid ''density'' ρ and fluid ''velocity'' $\boldsymbol{v} = \boldsymbol{\nabla}_\perp \psi$. Vortices can exist despite the potential nature of the ''fluid'' flow. Indeed, the Madelung transform is singular at points where $\rho=0$; these are just branch points where the real and imaginary parts of the field become zero, while the velocity circulation $\oint \boldsymbol{v}\cdot d\boldsymbol{l} = 2\pi m$, where the integration is done on a closed path enclosing the singular point, and the integer m is known as the topological ''charge'' of the vortex. Thus the vortex soliton is a topological structure; it can disappear only when annihilated by a vortex soliton of the opposite charge. The development of the transverse instability of a dark soliton has close parallels in hydrodynamics: for instance, the Kelvin-Helmholtz instability, which occurs when the boundary between two flows develops so-called ''vortex streets'' [14]. Since a dark solitary stripe does not carry any topological charge, it is evident that vortices have to be born with equal and opposite topological charges.

It is straightforward to show that e-p plasmas can support large amplitude dark solitons as well. In the general case (amplitude large, but subject to the condition $|A_\perp| < \gamma_g G_0$), we cannot construct analytic solutions even in one dimension. It is possible, however, to extract the general properties of the solution by using reasonably simple techniques, especially when $A_\perp$ has the time dependence

$$A(x,\tau) = \hat{A}(x)\exp(-i\lambda\tau), \tag{21}$$

where $A = A_\perp/\gamma_g G_0$ is the normalized amplitude, and is always less than unity. Here λ is so-called nonlinear frequency shift. This time dependence implies that the amplitude square is stationary (what follows, therefore, are classed as stationary solutions), and the dip of the wave does not propagate in the comoving frame with quite the group velocity of the linear wave.

The 1D equation (18) now can be cast in the form

$$\frac{d^2}{dx^2}\hat{A} + V'(\hat{A}) = 0, \tag{22}$$

where the prime on V denotes the derivative with respect to $\hat{A}$, and

$$V(\hat{A}) = (\lambda+2)\hat{A}^2 + 4\sqrt{1-\hat{A}^2} - 4 \tag{23}$$

denotes the potential. The resemblance of Eq. (22) to the one obeyed by a Newtonian particle in a nonlinear potential suggests an obvious method for analysis. One can easily prove that a bounded solution exists provided the nonlinear frequency shift is positive $(\lambda > 0)$. The profile of the potential, shown in Fig. 1 for $\lambda = 1$, reveals that the dark soliton solution may reside in the potential well. Equating $d^2\hat{A}/dx^2$ with zero, we estimate the upper bound on $\hat{A}$:

$$\hat{A}_{\mathrm{ub}} = \sqrt{1 - \left(\frac{2}{\lambda+2}\right)^2}. \tag{24}$$

The lowermost value of the amplitude $|A|$ is zero, that is, we recover the dark soliton. Note that for small values of λ $(\ll 1)$, $\hat{A}_{\mathrm{ub}} \to 0$, while $\hat{A}_{\mathrm{ub}} \to 1$ for $\lambda \gg 1$.

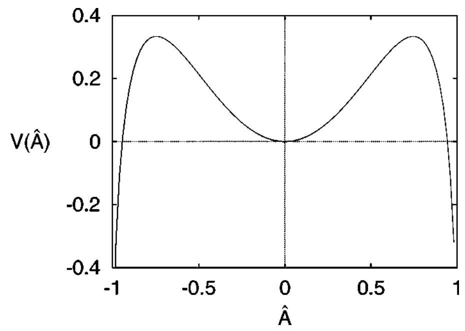

FIG. 1. The form of the potential $V(\hat{A})$ in Eq. (22) is shown for $\lambda=1$.

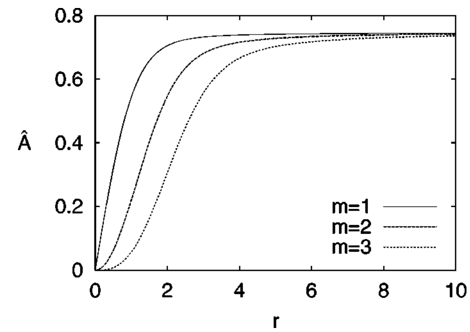

FIG. 2. The form of the solitonlike solutions for the normalized potential $\hat{A}$ is shown for $m=1$, 2, and 3 in the case of $\lambda=1$.

Similarly one can show the existence of vortex soliton solutions in two dimensions. We shall again seek stationary solutions in 2D polar coordinates (r,θ). The ansatz

$$A=\hat{A}(r)\exp(im\theta-i\lambda\tau), \tag{25}$$

with $\hat{A}(r)$ real, and with the perpendicular Laplacian operator given by

$$\nabla_{\perp}^{2}=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{m^2}{r^2}, \tag{26}$$

converts Eq. (17) to the ordinary differential equation [similar to Eq. (22)]

$$\frac{d^2}{dr^2}\hat{A}+V'(\hat{A})=-\frac{1}{r}\frac{d\hat{A}}{dr}+\frac{m^2}{r^2}\hat{A}, \tag{27}$$

where the potential $V(\hat{A})$ is the same as the one-dimensional expression given by Eq. (23). If we were to extend the ''particle in a potential'' analogy further, Eq. (27) could be viewed as the nonconservative motion of a particle. Since the right-hand side approaches zero in the limit $r\to\infty$, Eq. (27) gives precisely the 1D asymptotic value [Eq. (24) for the vector potential $\hat{A}$]. The behavior at the origin $(r=0)$ is totally different; the regular singular point at the origin $r=0$ forces the acceptable $\hat{A}$ to vanish for $m\geqslant 1$ as r^m. The numerical solutions of the two-dimensional nonlinear Schrödinger equation for $m=1$, 2, and 3 are shown in Fig. 2. As expected, the solitonlike solutions evidently go to zero as r^m for small r, and reach an m-independent asymptotic value predicted by Eq. (24).

More general aspects of the dynamics of the EM field can be studied mainly through numerical simulations of Eq. (17)—this is beyond the intended scope of this paper. We content ourselves here by making a few qualitative remarks, and pointing out directions for future efforts. The nonlinearity in Eq. (17) is a faster growing function (faster than the cubic) of the field amplitude, and does not exhibit saturation. One would expect, then, that the development of vortex chain structures from the dark stripe soliton instability for the general system will be faster in comparison with the low amplitude (cubic nonlinearity) case. This is, indeed, confirmed by a numerical solution of Eq. (17); the detailed results of the simulation will be discussed in a later publication. The stability of the vortex soliton solution is another issue that has to be dealt with. It is usually believed, however, that vortices with $m=\pm 1$ are topologically stable, whereas vortices with larger value of the ''charge'' m may decay into ''single-charge'' vortices.

In three dimensions the vortices form the so-called vortex line (it looks like a pancake with a hole in the center, hanging and moving along the wire-vortex line). Effects related to the finite group velocity dispersion may lead to a transverse instability of the vortex line. All these interesting effects are left for future studies.

We would like to emphasize that the electromagnetic fields associated with dark and vortex solitons are asymptotically nonvanishing (at infinity). Due to the generally accepted requirement that in physical system, the fields be localized in all directions, these objects have received much less attention than their localized cousins. However, in recent experiments studying laser field dynamics in different kinds of optical media, it was demonstrated that dark and vortex solitons can be readily created as superimpositions upon a localized field background [9]. This background can be just a few times wider than the soliton width. During propagation the background spreads out, reducing its own intensity. In light of these experiments let us try to put in perspective the current study of dark and vortex solitons in e-p plasmas in the early Universe. Because the typical scale length of these solitons is the collisionless skin depth, we would need a supporting background spanning several skin depths. This should pose no problem, because the ambient uniform field background could easily foot the bill. The next scale length on which we encounter ''bulletlike'' electromagnetic structures (which owe their origin to the baryon asymmetry) is considerably larger than the skin depth. Thus the dark and vortex solitons can propagate in a slowly changing background (spreading and decreasing in intensity with the diffractive spreading rate of the soliton decreasing as the background expands), adiabatically maintaining their properties, until they hit baryon-asymmetry scale lengths.

Topological considerations will insure the preservation of the singular points during propagation. In propagating vortex chains, the vortices can move away from one another, reducing the possibility of their mutual annihilation. The propagation introduces elements similar to Hubble expansion—the structures "run away" from one another. These highly speculative remarks need careful investigation. It is possible that the spreading of the background field may just affect the vortex distribution, and only a cosmological expansion will drive them apart.

What is extremely significant is that during the evolution of the fields, the integrals of motion should be preserved. It is easy to prove, by direct calculations, that Eq. (17) conserves the angular momentum $\boldsymbol{M}$:

$$(\boldsymbol{M})_z = \frac{i}{2}\int d\boldsymbol{r}_\perp [\boldsymbol{r}_\perp \times (A_\perp^* \boldsymbol{\nabla}_\perp A_\perp - \mathrm{c.c.})]_z . \qquad (28)$$

Equation (28) for the angular momentum is a paraxial approximation for the orbital angular momentum, $\boldsymbol{M}_E = \int d\boldsymbol{r}[\boldsymbol{r} \times (\boldsymbol{E}\times\boldsymbol{B})]$, of the EM field [16]. The angular momentum carried by the vortices is $M_z = mN$, where N is another conserved quantity known as the "photon number" $N = \int d\boldsymbol{r}_\perp |A_\perp|^2$ [17].

It follows, then, that relativistic e-p plasmas are capable of sustaining electromagnetic vortexlike structures, and that these structures have domains in which the EM fields carry nonzero angular momenta, although the total angular momentum of the entire system is zero. If this angular momentum could, somehow, be locally transferred to the surrounding medium, we would have a rather effective mechanism for imparting angular momentum to different domains of matter in the early universe. In our next publication we will show that when baryon asymmetry effects are incorporated, the medium can, indeed, acquire angular momentum from the EM field vortices.

IV. CONCLUSIONS

We have investigated the dynamics of the highly relativistic ($\gamma_g \gg 1$) nonlinear propagation of electromagnetic waves in unmagnetized hot electron-positron plasmas. The system is described by a nonlinear Schrödinger equation (17) with an inverse square root type (nonsaturating) nonlinearity. We have shown the possibility of dark and vortex soliton type solutions for this equation. The transverse instability of dark soliton stripes leads to the formation of a vortex chain such that the EM fields in each vortex carry angular momentum. Such objects could play an important role in cosmology as sources of the structure formation in the MeV epoch of the evolution of the Universe. In commonly adopted cosmological scenarios about the origin of the rotation of galaxies, structures grow in a hierarchy by the gravitational assembly of clumps out of subclumps. The origin of the angular momentum of galaxies, if they were formed from initial fluctuations in a Friedman Universe, was suggested (by Hoyle [18]) to be due to tidal interactions between the condensing system [4]. However, it is still not clear whether this mechanism gives an adequate solution [19]. We hope that the suggested mechanism of angular momentum generation in the MeV epoch of the Universe is an interesting alternative to explore and examine. Electromagnetism, operating through the versatile substrate of the e-p plasma, seems to readily generate these highly interesting, long-lived objects—the carriers of large amounts of mass, energy, and angular momentum. Since an initial localization of mass, energy, and angular momentum is precisely the seed that gravity needs for eventual structure formation, electromagnetism may have provided a key element in the construction of a large-scale map of the observable Universe.

Results of this paper can also be applied to astrophysical objects like pulsars, and active galactic nuclei—the e-p pairs are thought to be a major constituent of the plasma emanating both from the pulsars, and from the inner region of the accretion disks surrounding the central black holes.

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