A Shear-Flow Instability Relevant to Advanced Tokamak Operation

Tomoya TATSUNO and William DORLAND

Center for Scientific Computation and Mathematical Modeling, The University of Maryland, College Park, Maryland 20742-3289, USA

(Received 1 February 2006 / Accepted 17 February 2006)

An ideal magnetohydrodynamic instability which is driven by sheared flow and which may be relevant to reversed-shear advanced tokamak operation is described. If there is flow shear at the $q_{\min}$ surface, relatively weak velocity shear can drive this instability, with a time scale of the flow. The flows may be significantly slower than the ambient poloidal Alfvén velocity, and no inflection point is needed. Thus the time scale of the instability may be significantly longer than that of the poloidal Alfvén transit time, and it might account for disruption of reversed-shear discharges recently observed in JT-60U.

Keywords: MHD, shear flow, instability, reversed-shear tokamak, disruption

DOI: 10.1585/pfr.1.015

In recent advanced tokamak experiments, disruption is observed which is specific for reversed-shear configurations [1]. When $q_{\min}$ value approaches to a certain integral value, the plasma becomes unstable, and experiences disruption, which shows a longer than ideal but shorter than resistive time scale in some cases. In this paper, we present single helicity, nonlinear simulations of an ideal, low mode number, magnetohydrodynamic (MHD) instability that is driven by the combined effect of sheared flow and non-monotonic field profile. More details will be presented in a longer, separate paper. This instability is a good candidate for explaining the experimental observations. The growth rate of this instability is governed by the magnitude of the flow shear. It is larger than that of the typical resistive instability and smaller than that of the typical magnetic-field driven ideal instability. It is interesting to note that the flow profile we use here is itself stable without magnetic field, and any magnetic field profile is itself stable in slab geometry without background flow, so the instability is truly a combined effect of shear flow and magnetic field.

The instability we discuss here was first pointed out by Stern [2], who realized that an inhomogeneous magnetic field may break the conservation of vorticity, just as the inflection point does in the case of Kelvin-Helmholtz (KH) instability [3]. Kent [4] extended this idea to the case of symmetric flow profile, and obtained several instability conditions in terms of the background values at the edge. Later, Chen and Morrison [5] extended Kent's work. We review the understanding of this instability, present the first nonlinear simulations (in slab geometry), and point out that this instability may actually be relevant to reversed-shear discharge of the advanced tokamak.

Since the instability is global and depends strongly on the detail of the field profile, it is difficult to make a quantitative discussion by analytic calculation. We cannot discuss the instability in the infinite domain, since the infinite difference of the shear flow at both edges formally prevents exponential instability due to its stretching (shearing) effect [6]. On the other hand, a periodic domain would inevitably introduce inflection point (so as the case of finite flow difference in the infinite domain), which brings us confusion to distinguish the phenomena from KH instability [7]. In practice, the overall growth rate has never been obtained by analytic calculation even for a particular profile in the simplest slab geometry. Here we present the first linear and nonlinear results from numerical simulations in the slab geometry.

In order to simulate the incompressible motion of the plasma, we developed a pseudospectral code which solves two-dimensional reduced MHD equations with vorticity-stream function and flux function formulation. To resolve the nonperiodic dynamics in the radial direction with spectral techniques, we use a Chebyshev polynomial basis. Our simulations show that nonlinear evolution of the reversed-shear tokamak example may lead to disruption with a time scale slower than Alfvén time and faster than the resistive one, which might account for the disruption recently observed in JT-60U [1].

We discuss the incompressible motion of shear flow plasmas in slab geometry. Taking the variation of background field in x (radial) direction, we solve Strauss' reduced MHD equations with finite viscosity and resistivity in the single-helicity (two-dimensional) limit [8],

$$\partial_t \nabla_\perp^2 U + \boldsymbol{v} \cdot \nabla \nabla_\perp^2 U = \boldsymbol{B} \cdot \nabla \nabla_\perp^2 A + \nu \nabla_\perp^2 \nabla_\perp^2 U, \qquad (1)$$

author's e-mail: tatsuno@umd.edu

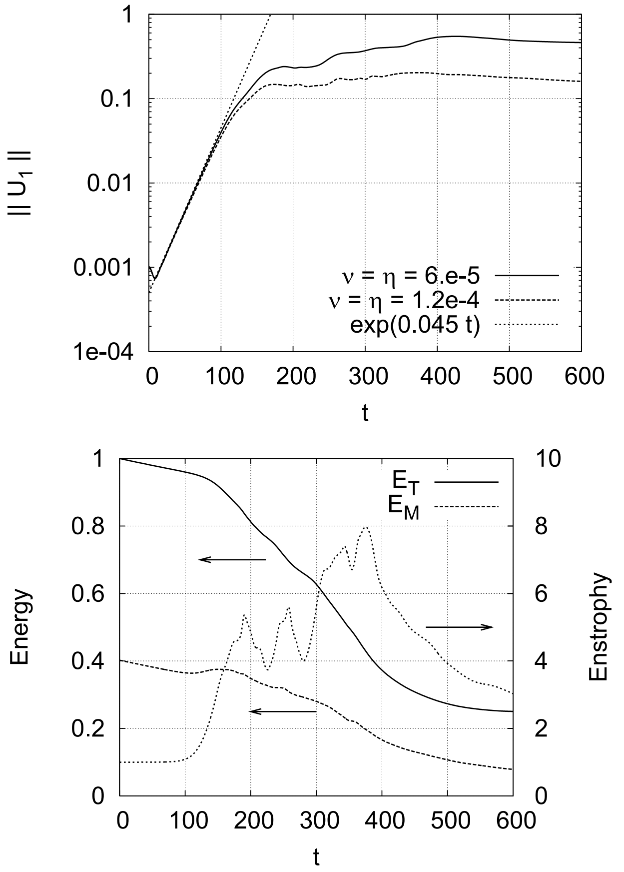

Fig. 1 Time evolutions of $||U_1||$, and enstrophy, total (E_{T}) and magnetic (E_{M}) energy normalized by their initial total values in the simulation with $\nu = \eta = 6 \times 10^{-5}$ and 768×768 grid points. Also shown is the evolution of $||U_1||$ with $\nu = \eta = 1.2 \times 10^{-4}$ and 512×512 grid points for comparison.

$$\partial_t A = \boldsymbol{B} \cdot \nabla U + \eta \nabla_\perp^2 A, \tag{2}$$

with stream function U and flux function A introduced by

$$\boldsymbol{v} = \nabla U \times \boldsymbol{e}_z, \qquad \boldsymbol{B} = \nabla A \times \boldsymbol{e}_z + B_z \boldsymbol{e}_z, \tag{3}$$

where space is normalized by half system size L in the radial direction, time is normalized by $L/v_{\mathrm{A}y}$, and $v_{\mathrm{A}y}$ denotes the Alfvén velocity defined by the y (poloidal) component of the magnetic field. The single-helicity calculation may be justified because there is only a single set of unstable low wave numbers in the example shown here. The inclusion of another dimension should not alter the qualitative results. Our boundary condition corresponds to the ideal, no-slip wall at $x = \pm 1$, and poloidal periodicity.

Figure 1 shows the time evolution of various integral quantities. The straight line in the evolution of the L^2 norm of perturbed stream function $||U_1||$ shows the exponential growth corresponding to the growth rate 0.045 obtained from an independent numerical eigenvalue analysis of the ideal ($\nu = \eta = 0$) system, which will be presented elsewhere. We also showed an evolution of $||U_1||$ with $\nu = \eta = 1.2 \times 10^{-4}$ for comparison, which shows good convergence of the linear growth rate with respect to diffusivity and resolution. The linear instability grows exponentially until $t \lesssim 100$. After this time, the system is fully nonlinear.

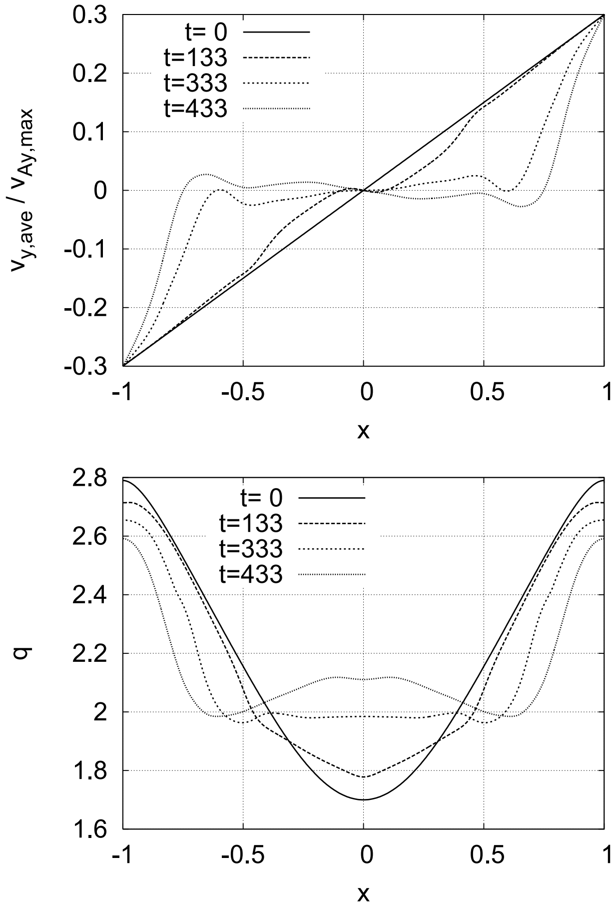

Fig. 2 Time evolution of the average flow and safety factor.

The time evolution of the average velocity and safety factor q are shown in Fig. 2. It should be stressed that the initial reversal of the magnetic shear is essential for the instability. Since JT-60U often experiences disruptions especially when $q_{\min}$ crosses 2, we assumed initial $q_{\min} = 1.7$ and considered the (2, 1) mode. As is seen from the figure, the instability tends to lower the q value just outside of $q \sim 2$ surfaces. At the same time, it raises the central $q(0)$. The instability doesn't alter the profile of velocity and safety factor in the outer region $|x| \gtrsim 0.6$, but it flattens and makes them approach zero in the central region in the linear regime.

When the instability comes into its nonlinear stage at $t \gtrsim 100$, the enstrophy grows rapidly, and shows a somewhat complicated behavior. The average velocity and q-profile at $t = 333$ are flattened in a fairly wide range, and are close to zero in $|x| \lesssim 0.5$. At $t \simeq 380$, when the enstrophy growth peaks, the q-profile is raised further, with $q > 2$ almost everywhere by $t = 433$.

Because of the global rearrangement of the current profile, we believe that the instability is a good candidate to explain some of the disruptive events observed in JT-60U. The necessary flow is about 20-40% of the poloidal Alfvén velocity, and the growth rate is one order of magnitude smaller than the flow shear rate in the example shown here.

We acknowledge fruitful discussions with Drs. Yasutomo Ishii and Nobuyuki Aiba. This research is in part supported by US DOE Center for Multiscale Plasma Dynamics.

[1] S. Takeji *et al*., Nucl. Fusion **42**, 5 (2002).
[2] M.E. Stern, Phys. Fluids **6**, 636 (1959).
[3] J.W.S. Rayleigh, Proc. London Math. Soc. **9**, 57 (1880).
[4] A. Kent, Phys. Fluids **9**, 1286 (1966); J. Plasma Phys. **2**, 543 (1968).
[5] X.L. Chen and P.J. Morrison, Phys. Fluids B **3**, 863 (1991).
[6] F. Volponi, Z. Yoshida and T. Tatsuno, Phys. Plasmas **7**, 2314 (2000); T. Tatsuno, F. Volponi and Z. Yoshida, Phys. Plasmas **8**, 399 (2001).
[7] A. Ito *et al*., Phys. Plasmas **9**, 4856 (2002).
[8] H.R. Strauss, Phys. Fluids **19**, 134 (1976).