Shear-flow induced stabilization of kinklike modes

Francesco Volponi[a)]
Graduate School of Engineering, The University of Tokyo, Tokyo 113-8656, Japan

Zensho Yoshida[b)] and Tomoya Tatsuno[c)]
Graduate School of Frontier Sciences, The University of Tokyo, Tokyo 113-8656, Japan

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The stabilizing effect of a shear flow on a current-driven instability has been studied by means of Kelvin's representation of spatially-inhomogeneous Galilean transform. Even though conspicuous transient growth may occur, the mixing effect of the shear flow overcomes the instability and damps kinklike modes. © *2000 American Institute of Physics.* [S1070-664X(00)01506-8]

I. INTRODUCTION

Shear flows represent a central feature and an open problem in various physical systems. The importance of the role of shear flows stems from a simple fact; a spatially inhomogeneous flow can drastically alter the modes of their evolution. In other words, depending on the physical characteristics of the system they are embedded in, sheared flows can lead to destabilization or stabilization. In the astrophysical context, for example, differential rotation is considered to be the priming factor of a dynamo process which might account for the presence of strong magnetic fields in planets, stars, and galaxies. In laboratory plasmas the interest in shear flows concentrates on their possible stabilizing influence on unstable modes of fusion plasmas. In spite of their importance, rigorous treatment of the effect of shear has always been deficient. This is primarily due to the formidable difficulties which plagued the analysis of the differential operators associated with shear flows. These operators indeed turn out to be non-self-adjoint and this fact entails the impossibility of resolving them in terms of orthogonal eigenfunctions. In recent years, however, the so-called nonmodal approach[1,2] is driving important progress in the study of shear flows both in ordinary hydrodynamics and in magnetohydrodynamics (MHD). The nonmodal method, which was introduced by Lord Kelvin[3] more than a century ago, consists of two basic methods to solve hyperbolic partial differential equations (PDEs); the characteristics method[4] and the Fourier expansion method (modal method). The solution to the characteristic ordinary differential equations (ODEs), associated with a shear flow $\mathbf{u}_0$, gives a new coordinate system (spatiotemporal) on which the convective (Lagrangian) derivative $(\partial_t+\mathbf{u}_0\cdot\boldsymbol{\nabla})$ reduces into a simple temporal derivative (∂_τ). In some special cases, the remaining spatial derivatives involved in the PDEs can be converted, by Fourier transform, into multiplications of some coefficients (which may be time dependent). These two procedures transform a complicated, spatially inhomogeneous system of PDEs into an easily manageable, temporally inhomogeneous system of ODEs.

A remarkable finding due to the application of this scheme was the so-called magnetorotational instability[5] in weakly magnetized accretion disks in astrophysical plasmas. There are several pioneering theories predicting very transient (neither exponential nor sinusoidal) behavior of plasmas driven by shear flows. In these theories, the free energy that drives the dynamics is primarily due to the shear flow. In real laboratory systems, as well as in space plasmas, some different free energies can stem from spatial and geometrical inhomogeneities of ambient physical quantities such as magnetic fields. One of the most important neglect committed in the simple one-dimensional treatment of the ambient fields (slab model) is the curvature of magnetic field lines, which yields the energy to drive the so-called "kink instability."

Although there are many phenomenological or approximate treatments of the kink modes under the influence of a shear flow, the basic relation between the shear flow and the instability remains unaddressed on a rigorous basis. The aim of this paper is to develop a theoretical foundation for the analysis of kinktype instabilities put in a shear flow.

We apply the nonmodal approach for a slab plasma model. As mentioned above, the naive formulation of a slab plasma drops the kink-mode driving term. However, invoking a standard technique of incorporating an equivalent magnetic-field curvature effect, we can introduce an artificial "kink-drive term" into the model. Physical applicability of the model is rather limited; only spatially localized behavior is within the scope. The theory, however, captures the essential nature of the competition between the flow shear and the instability driving effect. We will show that the velocity shear induces a time dependency on the wave vector, and the resultant "phase mixing" finally overcomes the instability drive.

In Sec. II we derive the model equations. The nonmodal method is applied in Sec. III. Results are presented and discussed in Sec. IV.

II. MODEL OF SHEAR-FLOW MHD

A. Ideal magnetohydrodynamics

Neglecting viscous and resistive effects the dynamics of a plasma is described by ideal MHD equations,

[a)]Electronic mail: fvolponi@plasma.q.t.u-tokyo.ac.jp
[b)]Electronic mail: yoshida@plasma.q.t.u-tokyo.ac.jp
[c)]Electronic mail: tatsuno@plasma.q.t.u-tokyo.ac.jp

$$\rho(\partial_t+\mathbf{u}\cdot\nabla)\mathbf{u}=\mathbf{j}\times\mathbf{B}-\nabla p, \tag{1}$$

$$\partial_t\mathbf{B}=\nabla\times(\mathbf{u}\times\mathbf{B}), \tag{2}$$

$$\partial_t\rho+\nabla\cdot(\rho\mathbf{u})=0, \tag{3}$$

where ρ and $\mathbf{j}$ are, respectively, the mass and current densities, $\mathbf{u}$ is the velocity and $\mathbf{B}$ is the magnetic field. The convective derivative $(\partial_t+\mathbf{u}\cdot\nabla)$ represents the temporal derivative in the frame of the fluid motion. Exploiting the relation $\mathbf{j}=\mu_0^{-1}\nabla\times\mathbf{B}$ (μ_0 is the vacuum permeability) and assuming a barotropic relation $\nabla p=c_s^2\nabla\rho$ (c_s is the sound speed) we can eliminate $\mathbf{j}$ and p from Eq. (1).

We introduce the following set of dimensionless variables:

$$\mathbf{x}=l\hat{\mathbf{x}},\quad \mathbf{B}=B_T\hat{\mathbf{B}},\quad \rho=\rho_0\hat{\rho},\quad t=(l/c_A)\hat{t},\quad \mathbf{u}=c_A\hat{\mathbf{u}}, \tag{4}$$

where l is a characteristic length scale of the system, B_T and ρ_0 are the representing values of the magnetic field and density, respectively, and $c_A=B_T/(\mu_0\rho_0)^{1/2}$ is the Alfvén speed. The dimensionless form of (1)–(3) is

$$\hat{\rho}(\partial_{\hat{t}}+\hat{\mathbf{u}}\cdot\nabla)\hat{\mathbf{u}}=(\nabla\times\hat{\mathbf{B}})\times\hat{\mathbf{B}}-\beta\nabla\hat{\rho}, \tag{5}$$

$$\partial_{\hat{t}}\hat{\mathbf{B}}=\nabla\times(\hat{\mathbf{u}}\times\hat{\mathbf{B}}), \tag{6}$$

$$\partial_{\hat{t}}\hat{\rho}+\nabla\cdot(\hat{\rho}\hat{\mathbf{u}})=0, \tag{7}$$

where $\beta=(c_s/c_A)^2$. In what follows we will drop the *hat* in order to simplify the notation.

We decompose the physical quantities into their equilibrium and small perturbative components to write

$$\mathbf{B}=\mathbf{B}_0+\tilde{\mathbf{B}},\quad \mathbf{u}=\mathbf{u}_0+\tilde{\mathbf{u}},\quad \rho=\rho_0+\tilde{\rho}, \tag{8}$$

where the subscript ''0'' denotes equilibrium fields, and the tilde, the perturbations fields. Linearizing (5)–(7), we obtain

$$\rho_0(\partial_t+\mathbf{u}_0\cdot\nabla)\tilde{\mathbf{u}}+\tilde{\rho}\mathbf{u}_0\cdot\nabla\mathbf{u}_0+\rho_0\tilde{\mathbf{u}}\cdot\nabla\mathbf{u}_0$$
$$=[(\nabla\times\tilde{\mathbf{B}})\times\mathbf{B}_0+(\nabla\times\mathbf{B}_0)\times\tilde{\mathbf{B}}]-\beta\nabla\tilde{\rho}, \tag{9}$$

$$\partial_t\tilde{\mathbf{B}}=\nabla\times(\tilde{\mathbf{u}}\times\mathbf{B}_0)+\nabla\times(\mathbf{u}_0\times\tilde{\mathbf{B}}), \tag{10}$$

$$\partial_t\tilde{\rho}=-\nabla\cdot(\rho_0\tilde{\mathbf{u}}+\tilde{\rho}\mathbf{u}_0). \tag{11}$$

Here, we remark the importance of the inhomogeneous ambient flow $\mathbf{u}_0$. The standard linear MHD theory assumes that $\mathbf{u}_0$=constant $(\simeq 0)$. Then, the generator of the dynamics becomes self-adjoint, and this fact allows us to apply the modal method (spectral analysis). When $\mathbf{u}_0$ is not constant, however, the system becomes non-self-adjoint, and this introduces profound difficulties in the spectral analysis of the ideal MHD operator.[6,7]

B. One-dimensional slab model

We consider a one-dimensional slab plasma. We express all quantities in Cartesian $x-y-z$ coordinates where, to compare with tokamak geometry, x parallels the radial, y the poloidal, and z the toroidal coordinates. The x is the direction in which the ambient fields vary. The perturbation are functions of x and y, while they are homogeneous with respect to z.

We assume that the equilibrium density is a positive constant and set $\rho_0=1$. To model a tokamak plasma, we assume $|B_{0_y}/B_{0_z}|=\epsilon\ll 1$, and set B_T (representative value of $\mathbf{B}$) $=B_{0_z}$. The equilibrium magnetic field is given by

$$\mathbf{B}_0=\begin{pmatrix}0\\-\epsilon\\1\end{pmatrix}. \tag{12}$$

The shear flow $\mathbf{u}_0$ is an incompressible flow given by

$$\mathbf{u}_0=\begin{pmatrix}0\\-Ax\\0\end{pmatrix}, \tag{13}$$

where A represents the strength of the velocity shear.

By assuming a symmetry $(\partial_z=0)$ and incompressibility $(\nabla\cdot\tilde{\mathbf{u}}=0)$, we may write

$$\tilde{\mathbf{B}}=\nabla\psi\times\nabla z+\tilde{B}_z\nabla z,\quad \tilde{\mathbf{u}}=\nabla\phi\times\nabla z+\tilde{u}_z\nabla z. \tag{14}$$

A strong toroidal field (B_{0_z}) allows us to assume $\tilde{B}_z=\tilde{u}_z=0$ (tokamak ordering). Then the perturbations are represented by two fields; ψ (z component of the vector potential) and ϕ (velocity streamfunction) that give

$$\tilde{\mathbf{B}}=\nabla\psi\times\nabla z,\quad \tilde{\mathbf{u}}=\nabla\phi\times\nabla z. \tag{15}$$

Under this assumption, the perturbations (15) obey a self-consistent system of linear MHD equations, which describes the shear Alfvénic mode. We note that this mode of perturbations is decomposed from other compressional modes (slow and fast modes) and forms a closed system. A kink instability, that is an unstable shear Alfvénic mode, becomes most unstable when it is decoupled from compressional modes that have positive energies.

Let us introduce the ''kink-drive term'' into the model. As mentioned above, we have to assume an artificial ''curvature'' of the magnetic field lines, which yields the energy to drive the kink mode.

In the evolution equation (9), this effect appears as the term $(\nabla\times\mathbf{B}_0)\times\tilde{\mathbf{B}}$. Although $\nabla\times\mathbf{B}_0=0$ for the present $\mathbf{B}_0$ [see (12)], we may introduce an artificial curvature by calculating the curl derivative in the cylindrical coordinates.

For a general vector field $\mathbf{A}$, we have

$$(\nabla\times\mathbf{A})_r=\frac{1}{r}\partial_\theta A_z-\partial_z A_\theta,$$

$$(\nabla\times\mathbf{A})_\theta=\partial_z A_r-\partial_r A_z,$$

$$(\nabla\times\mathbf{A})_z=\frac{1}{r}(\partial_r(rA_\theta)-\partial_\theta A_r).$$

Here x parallels the radial coordinate r, while y and z correspond to the poloidal (θ) and toroidal coordinates. Using these correspondences, we obtain

$$(\nabla\times\mathbf{B}_0)_x=0,$$

$$(\nabla\times\mathbf{B}_0)_y=0,$$

$$(\nabla\times\mathbf{B}_0)_z=-\frac{1}{x}\partial_x(x)=-\frac{1}{x}.$$

The expression for $(\nabla\times\mathbf{B}_0)\times\tilde{\mathbf{B}}$ becomes

$$(\nabla\times\mathbf{B}_0)\times\tilde{\mathbf{B}}=-\frac{1}{x}\nabla\psi.$$

After taking the **curl** of the momentum equation (9) and the $\mathbf{curl}^{-1}$ of Faraday Law (10) in Cartesian coordinates we obtain

$$\partial_t(\Delta\phi)-Ax\partial_y(\Delta\phi)=-\epsilon\partial_y(\Delta\psi)-\epsilon x^{-2}\partial_y\psi, \tag{16}$$

$$\partial_t\psi-Ax\partial_y\psi=-\epsilon\partial_y\phi, \tag{17}$$

where $\Delta=\partial_x^2+\partial_y^2$ is the Laplacian. The second term on the right-hand side of (16) brings about the kink (current-driven) instability.

III. NONMODAL REPRESENTATION

We introduce a spatio-temporal coordinate transform[8] from the fixed (Eulerian) reference frame $x-y-z$ to the local moving frame (Lagrangian) going with the mean shear flow,

$$\xi=x, \tag{18}$$

$$\eta=y+Axt, \tag{19}$$

$$\tau=t. \tag{20}$$

The above transformation represents the characteristic rays of the convective derivative operator $D/Dt\equiv\partial_t+\mathbf{u}_0\cdot\nabla$. Indeed it can be readily obtained from the characteristic ODE associated with D/Dt. Solving

$$\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\-Ax\end{pmatrix}, \tag{21}$$

subject to an initial condition

$$\begin{pmatrix}x\\y\end{pmatrix}(0)=\begin{pmatrix}\xi\\\eta\end{pmatrix}, \tag{22}$$

yields (18) and (19).

Equations (18), (19), (20) induce the following transformation of the partials:

$$\partial_x=\partial_\xi+At\partial_\eta, \tag{23}$$

$$\partial_y=\partial_\eta, \tag{24}$$

$$\partial_t=\partial_\tau+Ax\partial_\eta. \tag{25}$$

The convective derivative $D/Dt=\partial_t-Ax\partial_y$ reduces into a simple "temporal" derivative ∂_τ.

Equations (16) and (17) now read

$$\partial_\tau(\Delta'\phi)=-\epsilon[\partial_\eta(\Delta'\psi)-R_0^{-2}\partial_\eta\psi], \tag{26}$$

$$\partial_\tau\psi=-\epsilon\partial_\eta\phi, \tag{27}$$

where $\Delta'=\partial_\xi^2+(1+A^2t^2)\partial_\eta^2+2At\partial_\xi\partial_\eta$ is the Laplacian represented in the new coordinate system and R_0 is a constant number representing the curvature radius of the system. We note that the present model of field-line curvature assumes radially localized perturbations.

The system (26)–(27) is homogeneous with respect to the coordinates ξ and η. Fourier transform of ϕ and ψ, thus, yields "good quantum numbers" k_ξ and k_η. Let us write

$$\psi(\xi,\eta,\tau)=\hat{\psi}(\tau)e^{i(k_\xi\xi+k_\eta\eta)},\qquad \phi(\xi,\eta,\tau)=\hat{\phi}(\tau)e^{i(k_\xi\xi+k_\eta\eta)}. \tag{28}$$

Then, the system of PDEs (26) and (27) reduces into a system of ODEs,

$$\frac{d}{d\tau}([(k_\xi+k_\eta A\tau)^2+k_\eta^2]\hat{\phi})=-\epsilon ik_\eta([(k_\xi+k_\eta A\tau)^2+k_\eta^2]-R_0^{-2})\hat{\psi}, \tag{29}$$

$$\frac{d}{d\tau}\hat{\psi}=-\epsilon ik_\eta\hat{\phi}. \tag{30}$$

We point out that Eqs. (23), (24), (25), and (28) imply the following transformation for the wave numbers:

$$k_x=k_\xi+Ak_\eta\tau,\quad k_y=k_\eta, \tag{31}$$

where k_x and k_y are, respectively, the wave numbers in the x and y coordinates. Equations (31) clearly shows that the k_x varies with time, implying that the shear flow deforms the mode of perturbation.

After simple manipulations, we obtain the following second order ODE for $\hat{\psi}$,

$$\frac{d^2}{dT^2}\hat{\psi}+2a\frac{k_x(T)/k_y}{(k_x(T)/k_y)^2+1}\frac{d}{dT}\hat{\psi}+\epsilon^2\left[1-\frac{r_0^{-2}}{(k_x(T)/k_y)^2+1}\right]\hat{\psi}=0, \tag{32}$$

where $k_x(T)/k_y=\kappa_\xi+aT$ and T,a,r_0,κ_ξ are the normalized quantities defined by

$$T=\tau k_\eta,\quad a=A(k_\eta)^{-1},\quad r_0=R_0k_\eta,\quad \kappa_\xi=k_\xi k_\eta^{-1}. \tag{33}$$

Defining

$$\mu(T)=2a\frac{k_x(T)/k_y}{(k_x(T)/k_y)^2+1}=2a\frac{\kappa_\xi+aT}{(\kappa_\xi+aT)^2+1}, \tag{34}$$

$$\Omega^2(T)=\epsilon^2\left[1-\frac{r_0^{-2}}{(k_x(T)/k_y)^2+1}\right]=\epsilon^2\left[1-\frac{r_0^{-2}}{(\kappa_\xi+aT)^2+1}\right], \tag{35}$$

we write (32) as

$$\frac{d^2}{dT^2}\hat{\psi}+\mu(T)\frac{d}{dT}\hat{\psi}+\Omega^2(T)\hat{\psi}=0.$$

TABLE I. Relation between κ_ξ and $\mu(T)$.

Case	κ_ξ	$\mu(T)$ at small T	$\mu(T)$ at large T
A	+	+	+
B	−	−	+

IV. ANALYSIS OF THE EVOLUTION OF THE PERTURBATIONS

We pursue the analysis of Eq. (32) in three steps. First we will examine the effect of the kinklike driving term and the shear term separately and then we will investigate their combined action.

A. No shear flow (Alfvén wave and kink instability)

Assuming $a=0$ (zero flow), Eq. (32) becomes

$$\frac{d^2}{dT^2}\hat{\psi}+\epsilon^2\left(1-\frac{r_0^{-2}}{\kappa_\xi^2+1}\right)\hat{\psi}=0. \tag{36}$$

The curvature radius r_0 is the characteristic parameter that dominates the behavior of the solution. For sufficiently large r_0 (small curvature), i.e., $r_0^2>1/(\kappa_\xi^2+1)$, (36) describes an harmonic oscillator, which represents the Alfvén wave. For a small r_0 (large curvature), we may have an exponentially growing solution, representing the kink instability. The transition from a stable oscillation (Alfvén wave) to an unstable motion (kink instability) depends on the wave number κ_ξ (=const. if $a=0$); $r_0^2=1/(\kappa_\xi^2+1)$ is the critical number.

B. No kink drive (shear-flow damping)

In the absence of the kink-driving term ($r_0^{-2}=0$), Eq. (32) reduces into

$$\frac{d^2}{dT^2}\hat{\psi}+2a\,\frac{\kappa_\xi+aT}{(\kappa_\xi+aT)^2+1}\,\frac{d}{dT}\hat{\psi}+\epsilon^2\hat{\psi}=0. \tag{37}$$

The behavior of the solution depends on the sign of the coefficient $\mu(T)$ defined in (34). Invoking the analogy of (37) with Newton's equation of motion, we can interpret the second term on the right-hand side of (37) as a frictional force. A positive $\mu(T)$ damps the oscillation of $\hat{\psi}$ (Alfvén wave).

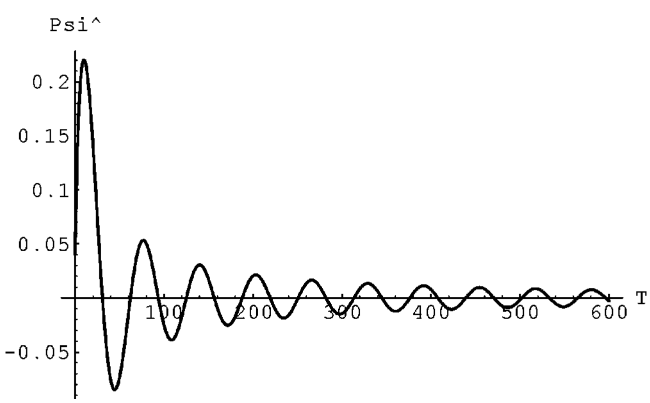

FIG. 1. Evolution of $\hat{\psi}$ for $r_0^{-2}=0$, $\kappa_\xi=4$, $\epsilon=0.1$, and $a=0.5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

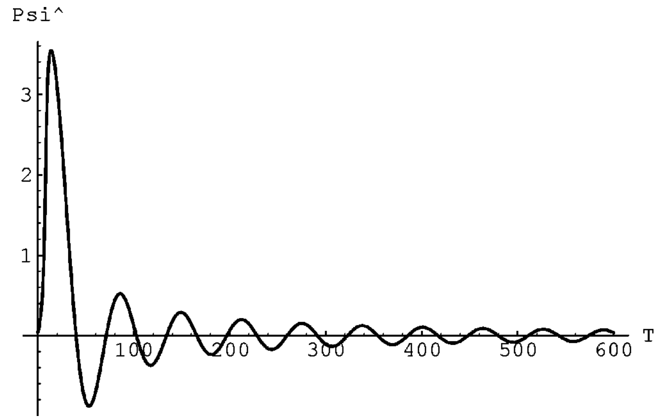

FIG. 2. Evolution of $\hat{\psi}$ for $r_0^{-2}=0$, $\kappa_\xi=-4$, $\epsilon=0.1$, and $a=0.5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

This behavior of the solution, which is due to the phase mixing effect induced by the shear flow, presents an analogy of the Landau damping that is induced by the shear flow in the coordinate-velocity phase space of the kinetic theory. On the contrary, a negative value of $\mu(T)$ yields amplification of $\hat{\psi}$.

The frictional coefficient $\mu(T)$ depends on the parameters κ_ξ and a. In the following analysis, without loss of generality, we will take the signs of A and k_η to be positive (then a is positive). In Table I we summarize the dependence of $\mu(T)$ on the sign of κ_ξ. We observe that, in any case, $\mu(T)$ is positive for large T. This implies that the solution of (37) is finally damped. In Case A, $\hat{\psi}$ shows only damping oscillations (Fig. 1), however, in Case B, a transient amplification of $\hat{\psi}$ occurs (Fig. 2). The maximum of the amplitude is reached around the time when $\mu(T)$ changes its sign.

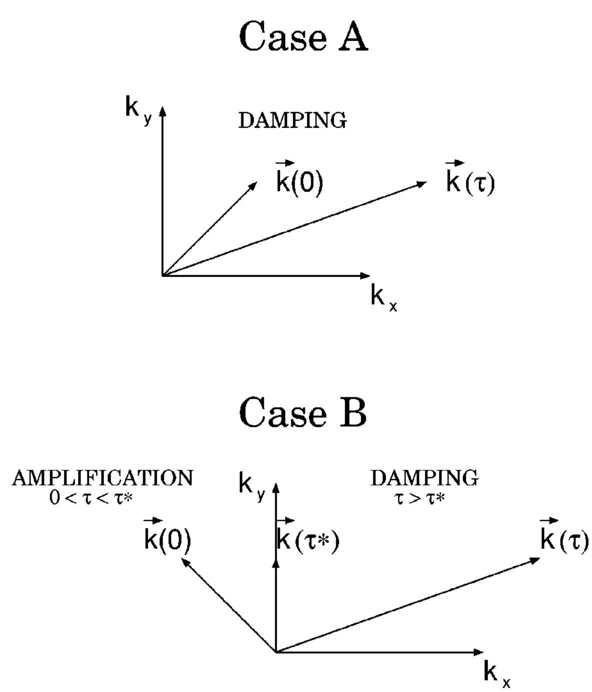

FIG. 3. Evolution of **k** for positive and negative values of k_ξ.

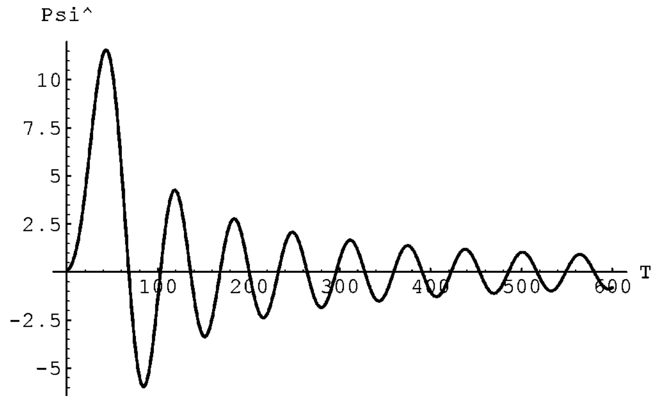

FIG. 4. Evolution of $\hat{\psi}$ for $r_0^{-2}=400$, $\kappa_\xi=4$, $\epsilon=0.1$, and $a=0.5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

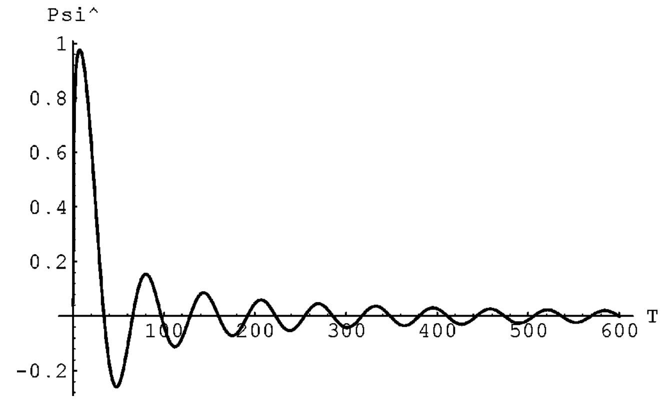

FIG. 6. Evolution of $\hat{\psi}$ for $r_0^{-2}=400$, $\kappa_\xi=-4$, $\epsilon=0.1$, and $a=5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

Let us see the evolution of the wave vector **k** (in the $x-y$ coordinates) and its relation to the behavior of $\hat{\psi}$. The initial configuration is $\mathbf{k}(0)=(k_\xi,k_\eta)$. First we consider the case when k_ξ is positive. By (31) we find that k_x increases monotonically with time, i.e., **k** is stretched in the positive k_x direction (Case A of Fig. 3). This case corresponds to the simple damping oscillations. When k_ξ is negative, the evolution of **k** experiences two distinct phases. For $0\leqslant\tau\leqslant\tau^*=-\kappa_\xi/a$, k_x shrinks until it becomes zero, and during this phase, the amplification of the perturbation proceeds. For $\tau\geqslant\tau^*$, the absolute value of k_x grows and **k** is stretched in the positive k_x direction (Case B of Fig. 3). This stretch of k_x yield the phase mixing damping.

C. Competition between the kink drive and the shear-flow induced damping

The discussion in the previous subsections shows the following relations:

$\Omega^2(T)>0$: oscillatory behavior,

$\Omega^2(T)<0$: instability,

$\mu(T)>0$: damping, $\mu(T)<0$: amplification.

We have seen that the kink-driving term (proportional to r_0^{-2}) contributes negatively in $\Omega^2(T)$. The shear-flow effect (proportional to a) diminishes the magnitude of the kink-driving term (for large T, it becomes proportional to T^{-2}). On the other hand, $\mu(T)$ becomes positive for large T. Therefore, the phase mixing effect induced by the shear flow is asymptotically stronger than the kink mode destabilization.

Let us study the behavior of the solution of (32) in more detail. Consider a small value of r_0 (high r_0^{-2}), so that the mixing damping is negligible. The solution grows until $\Omega^2(T)$ becomes positive. Writing $\Omega^2(\overline{T})=0$, this critical time $\overline{T}$ is given by the positive root of

$$\overline{T}=\frac{-\kappa_\xi\pm\sqrt{r_0^{-2}-1}}{a}. \tag{38}$$

We can then single out in the time domain an instability region $[0,\overline{T}]$ of growth for the oscillations.[1] As shown in Figs. 4 and 5, the growth can be very rapid, especially for negative k_ξ. Equation (38) suggests that a strong shear can diminish the time interval of the growth, resulting in a reduction of the maximum amplitude. This phenomenon is shown in Figs. 6 and 7 where we compare two different shears. The strong stabilizing effect of the shear is clearly visible. In Figs. 8 and 9 we consider two different configurations of the magnetic field. The results suggest that for small values of ϵ

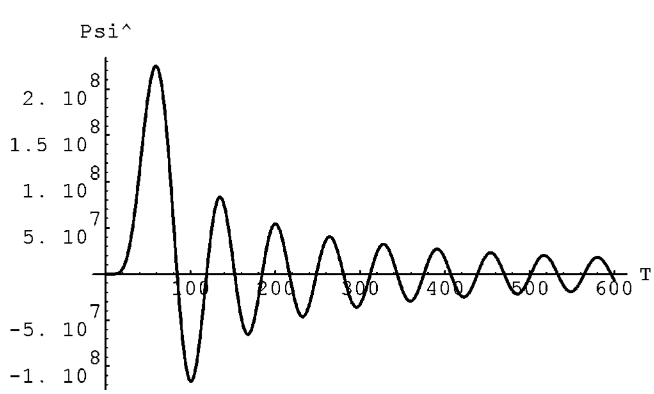

FIG. 5. Evolution of $\hat{\psi}$ for $r_0^{-2}=400$, $\kappa_\xi=-4$, $\epsilon=0.1$, and $a=0.5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

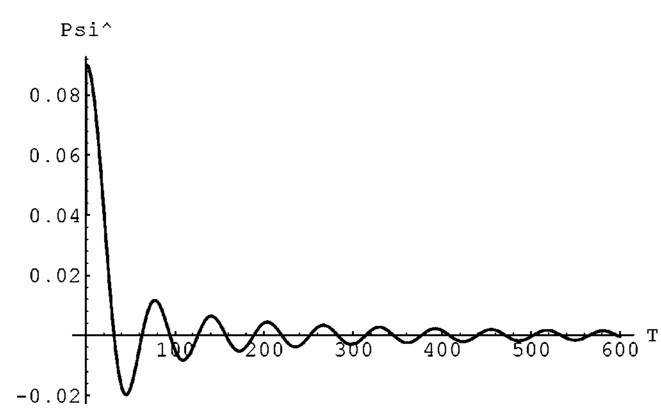

FIG. 7. Evolution of $\hat{\psi}$ for $r_0^{-2}=400$, $\kappa_\xi=-4$, $\epsilon=0.1$, and $a=50$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

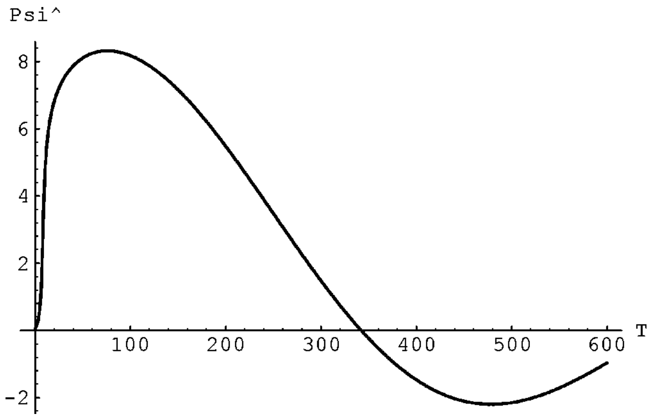

FIG. 8. Evolution of $\hat{\psi}$ for $r_0^{-2}=400$, $\kappa_\xi=-4$, $\epsilon=0.01$, and $a=0.5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

(i.e., $\mathbf{B}_0$ and $\mathbf{u}_0$ close to a condition of orthogonality) the kinklike modes are more effectively stabilized.

V. SUMMARY

A shear flow brings about the strong stabilizing effect that can overcome the kink-type instability at a sufficiently large time. The analysis of this process requires a nonmodal method. This stabilizing effect is due to the deformation (stretching) of the mode of the instability. To highlight this point, let us compare the above-mentioned results with the calculations for a rigid (nonsheared) flow. For a rigid flow $(0,-A,0)$ the transformation to a Lagrangian system is given by a Galilean transform,

$$\xi=x, \quad \eta=y+At, \quad \tau=t, \tag{39}$$

which transforms the partial derivatives as

$$\partial_x=\partial_\xi, \quad \partial_y=\partial_\eta, \quad \partial_t=\partial_\tau+A\partial_\eta. \tag{40}$$

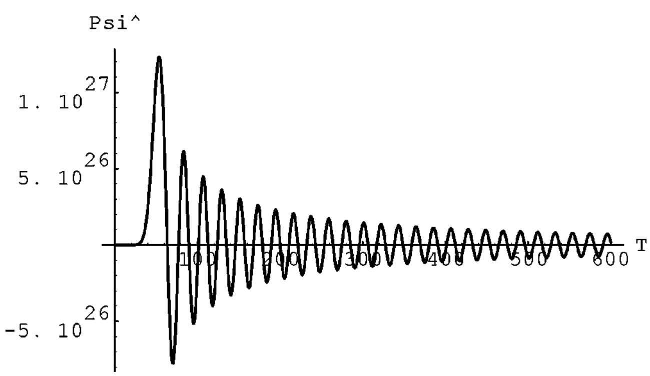

FIG. 9. Evolution of $\hat{\psi}$ for $r_0^{-2}=400$, $\kappa_\xi=-4$, $\epsilon=0.3$, and $a=0.5$. As initial values we have chosen $\hat{\psi}(0)=0.04$ and $(d/dT)\hat{\psi}(0)=0.05$.

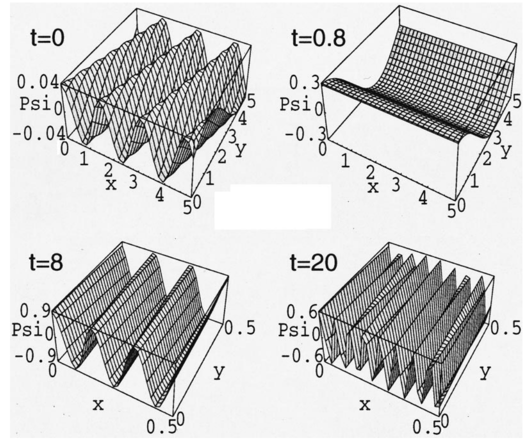

FIG. 10. Spatial dependence of the real part of the perturbation ψ relative to $\hat{\psi}$ in Fig. 6 at four different times.

Thus, for a rigid flow, the coordinate transform does not induce any time dependence of $\mathbf{k}$. In Fig. 10, we show the spatial profile of the real part of ψ (relative to the simulation in Fig. 6) at four different times. We observe that the mode is strongly deformed by the shear flow.

The evolution of a non-self-adjoint system is generally very complicated, and can be very different from usual oscillatory or exponential behavior. Transient amplification of perturbations may occur in an essentially stable system.[9] On the contrary, the mixing effect may induce a strong damping effect that dominates the long-term behavior of the system. Even though conspicuous transient growth for the perturbation fields are observed, an increase in the magnitude of the shear has a huge impact on the reduction of their maximum amplitude.

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[1] G. D. Chagelishvili, A. D. Rogava, and D. T. Tsiklauri, Phys. Plasmas **4**, 1182 (1997).
[2] G. D. Chagelishvili, A. G. Tevzadze, G. Bodo, and S. S. Moiseev, Phys. Rev. Lett. **79**, 3178 (1997).
[3] Lord Kelvin, Philos. Mag. **24**, 188 (1887).
[4] R. Courant and D. Hilbert, *Methods of Mathematical Physics* (Interscience, New York, 1962), Vol. II.
[5] S. A. Balbus and J. H. Hawley, Appl. J. **400**, 610 (1992).
[6] L. N. Trefethen, A. E. Trefethen, S. C. Reddy, and T. A. Driscoll, Science **261**, 578 (1993).
[7] S. C. Reddy, P. J. Schmid, and D. S. Henningson, SIAM (Soc. Ind. Appl. Math.) J. Appl. Math. **53**, 15 (1993).
[8] W. O. Criminale and P. G. Drazin, Stud. Appl. Math. **83**, 123 (1990).
[9] G. D. Chagelishvili, T. S. Hristov, R. G. Chanishvili, and J. G. Lominadze, Phys. Rev. E **47**, 366 (1993).